Port-Hamiltonian Systems for Fluid and Structural mechanics: Part II.

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Overview

Introduction

Fluid-structure system with piezo actuators

Discretization

Preliminary results

Limitations and further work
The port-Hamiltonian formalism brings together:

- Port-based modeling approach (bond-graph);
- Geometric mechanics (coordinate-free);
- Systems and control theory.
Bond graphs provide a unified framework for the modeling of physical systems

Henry Paynter (1959)
- Different domains (mechanical, electrical, hydraulic, thermal)
- Energy is the *lingua franca*;
- Complex systems are written as a composition of ideal components: energy-storage, energy-dissipation, energy-routing, etc.
Bond graph vs block diagram

Block diagram

```
input
Engine → torque → Wheel → angular speed
input
Engine → angular speed → Wheel → torque
```

Bond Graph

```
Engine \frac{torque}{angular speed} Wheel
```

“Bonds” are energy-conserving and acausal!
Bond graphs provides a unified language for systems of different physical domains.
Bond graphs are used to model COMPLEX systems.

Many commercial software based on bond-graphs:

- AMEsim
- 20-sim (U. Twente)
- Simscape (Matlab/Simulink)
- Dymola (Catia / Dassault)
Port-Hamiltonian systems = bond graph elements + Hamiltonian

Storage function is the Hamiltonian (energy) $H(x)$.

Storage flow variable: $f_c = -\dot{x}$

Storage effort variable: $e_c = \frac{\partial H}{\partial x}(x)$

Energy flow: $\dot{H}(t) = \frac{\partial^T H}{\partial x} \dot{x} = -e_c^T f_c = e_p^T f_p + e_r^T f_r$
Simple example of port-Hamiltonian system

From Newton 2nd Law:

\[ m\ddot{x} + kx = F_{\text{ext}}, \]

System energy:

\[ E = \frac{m\dot{x}^2}{2} + \frac{kx^2}{2} \]

By choosing \( p = m\dot{x} \) then \( H(p, x) = \frac{p^2}{2m} + \frac{kx^2}{2} \)

\[
\begin{bmatrix}
\dot{p} \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
mkx
\end{bmatrix}
+ 
\begin{bmatrix}
1 \\
0
\end{bmatrix} F_{\text{ext}},
\]

\[
\dot{x} =
\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
mkx
\end{bmatrix}
\]
Simple example of port-Hamiltonian system

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\[
\begin{bmatrix}
\dot{p} \\
\dot{x}
\end{bmatrix} = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
\partial_p H \\
\partial_x H
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} F_{ext},
\]

\[
\begin{bmatrix}
\dot{p} \\
\dot{x}
\end{bmatrix} = \begin{bmatrix}
1 & 0
\end{bmatrix} \begin{bmatrix}
\partial_p H \\
\partial_x H
\end{bmatrix} e_p, \quad \implies \dot{H} = F_{ext}\dot{x}
\]

\( F_{ext} \) and \( \dot{x} \) are the interconnection ports.
Typical mathematical representation of finite-dimensional port-Hamiltonian systems

\[ \dot{x} = J \frac{\partial H}{\partial x} + Bu, \]
\[ y = B^T \frac{\partial H}{\partial x}. \]

where:
- \( H(x) \): system Hamiltonian;
- \( x \in \mathbb{R}^n \): energy variables;
- \( u \in \mathbb{R}^m \): inputs;
- \( y \in \mathbb{R}^m \): outputs;
- \( J \): interconnection matrix (skew-symmetric);

\[ \implies \dot{H} = y^T u \]
The interconnection of two (N) PHS is still a PHS

**Individual systems**

\[
\begin{align*}
\dot{x}_1 &= J_1 \frac{\partial H_1}{\partial x_1} + B_1 u_1, \\
y_1 &= B_1^T \frac{\partial H_1}{\partial x_1}, \\
\dot{x}_2 &= J_2 \frac{\partial H_2}{\partial x_2} + B_2 u_2, \\
y_2 &= B_2^T \frac{\partial H_2}{\partial x_2},
\end{align*}
\]

**Interconnection**

\[
\begin{align*}
u_1 &= y_2 + u_e, \\
u_2 &= -y_1
\end{align*}
\]

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
J_1 & B_1 B_2^T \\
-B_2 B_1^T & J_2
\end{bmatrix}
\begin{bmatrix}
\frac{\partial H}{\partial x_1} \\
\frac{\partial H}{\partial x_2}
\end{bmatrix}
+\begin{bmatrix}
B_1 \\
0
\end{bmatrix} u_e,
\]

\[
y_e = \begin{bmatrix}
B_1 \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial H}{\partial x_1} \\
\frac{\partial H}{\partial x_2}
\end{bmatrix}
\]

\[
\frac{dH}{dt} = y_e^T u_e
\]
**Ex.: Levitating iron ball in a magnetic field**

- $x_1 = \phi$: total magnetic flux
- $x_2 = y$: displacement of the ball
- $x_3 = mv = p$: kinetic momentum

\[
\begin{align*}
\frac{dx}{dt} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial x} = i \\
H &= \frac{x_1^2}{2L(x_2)} - mgx_2 + \frac{x_3^2}{2m}, \\
\dot{H} &= ui.
\end{align*}
\]

- $\partial_{x_1} H$: current through the coil
- $\partial_{x_2} H$: velocity of the ball
- $\partial_{x_3} H$: electro-motive + gravity force
PHSs are convenient for (non-linear) control design

Typical port-Hamiltonian representation:

\[
\dot{x} = J \frac{\partial H}{\partial x}(x) + Bu, \\
y = B^T \frac{\partial H}{\partial x}(x).
\]

We’ve seen that: \( \dot{H} = y^T u \)

What happens if \( u = -K(x)y, \) with \( K(x) > 0? \)

\[
\dot{H} = -y^T K(x)y \leq 0,
\]

If \( H(x) \) is lower bounded, the controlled system is stable!
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Fluid-structure excited by piezoelectric actuators

- Very flexible plate
Fluid-structure excited by piezoelectric actuators

- Very flexible plate
- Partially filled tank

= fluid/structure interactions
Fluid-structure excited by piezoelectric actuators

Proposition:

- Use piezoelectric patches as actuators (or sensors)
- Reduce vibrations and liquid sloshing by feedback

Problems:

- How to model the experiment?
- How to design a *robust* control law?
The problem can be decomposed into several subsystems that exchange energy.

Some reasons for using PHS:
- Multi-domain;
- Power-conserving;
- Object-oriented approach (modularity);
The beam can be written in PHS form (1)

\[ \mu \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial z} \left( EI \frac{\partial^2 w}{\partial z^2} \right) = 0, \]

boundary conditions:

- Fixed end \((z = 0)\):
  \( w(0, t) = 0 \) and \( \frac{\partial w}{\partial z}(0, t) = 0 \)

- Free end \((z = L)\):
  \( EI \frac{\partial^3 w}{\partial z^3} = \) tip force and
  \( EI \frac{\partial^2 w}{\partial z^2} = \) tip moment
The beam can be written in PHS form (2)

\[
\mu \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial z} \left( EI \frac{\partial^2 w}{\partial z^2} \right) = 0,
\]

Euler-Bernoulli

Port-Hamiltonian version

Defining: \( x_1 := \mu \frac{\partial w}{\partial t} \) and \( x_2 := \frac{\partial^2 w}{\partial z^2} \):

\[
\frac{\partial x_1}{\partial t} = - \frac{\partial^2}{\partial z^2} \left( EI x_2 \right), \\
\frac{\partial x_2}{\partial t} = \frac{\partial^2}{\partial z^2} \left( \frac{x_1}{\mu} \right).
\]

Energy function: \( H = \frac{1}{2} \int_{z=0}^{L} \left( \frac{x_1^2}{\mu} + EI x_2^2 \right) dz \)

\( \text{Kinetic energy} \) \hspace{1cm} \( \text{elastic energy} \)
The beam can be written in PHS form (2)

Euler-Bernoulli

\[ \mu \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial z} \left( EI \frac{\partial^2 w}{\partial z^2} \right) = 0, \]

Port-Hamiltonian version

Defining: \( x_1 := \mu \frac{\partial w}{\partial t} \) and \( x_2 := \frac{\partial^2 w}{\partial z^2} \):

\[ \begin{align*}
\frac{\partial x_1}{\partial t} &= -\frac{\partial^2}{\partial z^2} (EIx_2), \\
\frac{\partial x_2}{\partial t} &= \frac{\partial^2}{\partial z^2} \left( \frac{x_1}{\mu} \right).
\end{align*} \]

\[ \begin{bmatrix}
\frac{\partial x_1}{\partial t} \\
\frac{\partial x_2}{\partial t}
\end{bmatrix} = 
\begin{bmatrix}
0 & -\frac{\partial^2}{\partial z^2} \\
-\frac{\partial^2}{\partial z^2} & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\delta H}{\delta x_1} \\
\frac{\delta H}{\delta x_2}
\end{bmatrix} + \begin{bmatrix}
f_c \\
e_c
\end{bmatrix}. \]

Energy function: \( H = \frac{1}{2} \int_{z=0}^{L} \left( \frac{x_1^2}{\mu} + EIx_2^2 \right) dz \)

- Kinetic energy
- Elastic energy
The beam can be written in PHS form (3)

Port-Hamiltonian version

\[ x_1 := \mu \frac{\partial w}{\partial t} \quad \text{and} \quad x_2 := \frac{\partial^2 w}{\partial z^2} \]

\[
\begin{bmatrix}
\frac{\partial x_1}{\partial t} \\
\frac{\partial x_2}{\partial t}
\end{bmatrix} =
\begin{bmatrix}
0 & -\frac{\partial^2}{\partial z^2} \\
\frac{\partial^2}{\partial z^2} & 0
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\]

\[-f_c \quad J \quad e_c
\]

The energy flow becomes:

\[
\dot{H} = \int_{z=0}^{L} e_1 \dot{x}_1 + e_2 \dot{x}_2 \, dz = \int_{z=0}^{L} -e_1 \frac{\partial^2 e_2}{\partial z^2} + e_2 \frac{\partial^2 e_1}{\partial z^2} \, dz
\]

\[
= \int_{z=0}^{L} \frac{\partial e}{\partial z} \left( -e_1 \frac{\partial e_2}{\partial z} + e_2 \frac{\partial e_1}{\partial z} \right) \, dz = \begin{bmatrix}
e_1 \\
e_2 \\
e_\partial 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
e_1 \quad e_\partial 2 \quad e_2 \quad e_\partial 1
\end{bmatrix}^L_{z=0}
\]

\[
e_1 = \frac{\delta H}{\delta x_1} = EI \frac{\partial^2 w}{\partial z^2} : \text{moment}
\]

\[
e_2 = \frac{\delta H}{\delta x_2} = \frac{\partial w}{\partial t} : \text{speed}
\]

\[
e_\partial 1 = \frac{\partial}{\partial z} \frac{\delta H}{\delta x_1} = \frac{\partial}{\partial z} (EI \frac{\partial^2 w}{\partial z^2}) : \text{force}
\]

\[
e_\partial 2 = \frac{\partial}{\partial z} \frac{\delta H}{\delta x_2} = \frac{\partial}{\partial z} \frac{\partial w}{\partial t} : \text{angular speed}
\]
The fluid can be written in PHS form (1)

1D Saint-Venant

Hypothesis

- Incompressible fluid;
- Shallow water;
- Only hydrostatic pressure: $P = \rho gh$

Equations

\[
\frac{\partial h}{\partial t} = - \frac{\partial}{\partial x} (hu)
\]
\[
\frac{\partial u}{\partial t} = - u \frac{\partial u}{\partial x} - g \frac{\partial}{\partial x} h
\]

- $u$: fluid speed;
- $h$: fluid height;
The fluid can be written in PHS form (2)

\[
\rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u^2}{2} \right) = -g \frac{\partial h}{\partial x} \quad \text{(Navier-Stokes)},
\]

\[
\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x} (hu) \quad \text{(Mass conservation)}.
\]

with energy given by:

\[
H(u, h) = \frac{1}{2} \int_{x=-a/2}^{a/2} \left( \rho bu^2 h + \rho bh^2 \right) dx.
\]

- kinetic energy
- potential energy
The fluid can be written in PHS form (2)

\[ \rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u^2}{2} \right) = -g \frac{\partial h}{\partial x} \]  
(Navier-Stokes),

\[ \frac{\partial h}{\partial t} = - \frac{\partial}{\partial x} (hu) \]  
(Mass conservation).

with energy given by:

\[ H(u, h) = \frac{1}{2} \int_{x=-a/2}^{a/2} \left(\rho bu^2 h + \rho bgh^2 \right) dx.\]

\[ \frac{\partial h}{\partial t} = - \frac{\partial}{\partial x} (hu) = - \frac{\partial}{\partial x} \left( \frac{1}{\rho b} \frac{\delta H}{\delta h} \right) \]

\[ \frac{\partial u}{\partial t} = - \frac{\partial}{\partial x} \left( \frac{u^2}{2} + gh \right) = - \frac{\partial}{\partial x} \left( \frac{1}{\rho b} \frac{\delta H}{\delta h} \right) \]
The fluid can be written in PHS form (3)

Saint-Venant 1D: written in PH form

Defining: $\alpha_1 := \rho u$ and $\alpha_2 := bh$:

$$\frac{\partial}{\partial t} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 & -\partial_x \\ -\partial_x & 0 \end{bmatrix} \begin{bmatrix} \frac{\delta H}{\delta \alpha_1} \\ \frac{\delta H}{\delta \alpha_2} \end{bmatrix}$$

$$H(\alpha_1, \alpha_2) = \frac{1}{2} \int_{x=-a/2}^{a/2} \left( \frac{\alpha_1^2 \alpha_2}{\rho} + \rho g \left( \frac{\alpha_2^2}{2} \right) \right) dx.$$ 

where:

$$e_1^F = \frac{\delta H}{\delta \alpha_1} = bh u = \text{volumetric flow}, \quad e_2^F = \frac{\delta H}{\delta \alpha_2} = \rho \frac{u^2}{2} + \rho gh = \text{total pressure}$$

$$\frac{\partial H^F}{\partial t} = e_1^F(-a/2, t) e_2^F(-a/2, t) - e_1^F(a/2, t) e_2^F(a/2, t)$$

- vol. flow
- pressure
- vol. flow
- pressure
The tank can be written in PHS form

2nd Newton Law: \[ m_{RB} \ddot{w}_B(t) = F_{ext}, \]

Port-Hamiltonian equations

\[
\frac{\partial}{\partial t} \begin{bmatrix} p \\ w_B \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \partial_p H^{RB} \\ \partial_{w_B} H^{RB} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F_{ext},
\]

\[
\dot{w}_B = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \partial_p H^{RB} \\ \partial_{w_B} H^{RB} \end{bmatrix}
\]

where the Hamiltonian is equal to the kinetic energy: \( H^{RB}(p) = \frac{1}{2} \frac{p^2}{m_{RB}} \), and its rate of change is given by:

\[
\dot{H}^{RB} = \dot{w}_B \dot{p} = \dot{w}_B F_{ext}.
\]
We can interconnect each component through their ports

Total Hamiltonian: \( H = H^B + H^F + H^{RB} \)

Kinematic constraint (equal speed):

\[
\dot{w}_B = e_2^B(L, t) = e_1^F(-a/2, t)/S = e_1^F(a/2, t)/S
\]

Sum of forces equal to zero:

\[
\mathcal{F} = -\frac{\partial}{\partial z} e_1^B(L, t) + F_{\text{ext}} e_2^F(a/2, t) S + e_2^F(-a/2, t) S = 0
\]

\[\Rightarrow \dot{H} = \dot{H}^B + \dot{H}^F + \dot{H}^{RB} = \dot{w}_B \mathcal{F} = 0\]
In order to simulate, we have to approximate infinite-dimensional equations

**Spatial discretization methods:**

- Mixed finite-elements: Golo 2003
  - Port-Hamiltonian structure is conserved after discretization;
  - No numerical dissipation;
  - Boundary conditions \( e_p \) appears as interconnection ports
- Pseudo-spectral spatial symplectic reduction: Moulla 2012
Discretization methods leads to finite-dimensional PHS

\[
\begin{bmatrix}
\frac{\partial x_1}{\partial x_2}
\end{bmatrix}
= \begin{bmatrix}
0 & -\frac{\partial^2}{\partial z^2} \\
\frac{\partial^2}{\partial z^2} & 0
\end{bmatrix}
\begin{bmatrix}
\delta x_1 H \\
\delta x_2 H
\end{bmatrix}
\]

\[
\dot{x} = J \frac{\partial H^d}{\partial x} + Bu,
\]

\[
y = B^T \frac{\partial H^d}{\partial x} + Du.
\]

\[
u = \begin{bmatrix}
\e_1^B(L) & \e_{\partial 1}^B(L) & \e_2^B(0) & \e_{\partial 2}^B(0)
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
-\e_2^B(L) & \e_2^B(L) & -\e_{\partial 1}^B(0) & \e_1^B(0)
\end{bmatrix}
\]

\[
\dot{H} = \dot{H}^d = y^T u
\]
Once we’ve discretized each subsystem, we find a set of DAEs:

\[
\dot{x}(t) = J_d \frac{\partial H_d}{\partial x} + Bu, \\
y^i = (B)^T \frac{\partial H_d}{\partial x} + Du,
\]

Final discrete finite-dimensional system:

\[
\dot{x}(t) = J_d \frac{\partial H_d}{\partial x} + Bu, \\
y^i = (B)^T \frac{\partial H_d}{\partial x} + Du,
\]

where:

\[
u = \begin{bmatrix} e^B_1 (L) & F^{RB}_{ext} & e^F_1 (-a/2) & e^F_2 (a/2) \end{bmatrix}^T
\]

\[
y = \begin{bmatrix} e^B_2 (L) & \dot{w}^{RB}_B & e^F_2 (-a/2) & e^F_1 (a/2) \end{bmatrix}^T
\]

\[
\mathcal{M} y + \mathcal{N} u = 0,
\]

Coupled system: Differential Algebraic Equations:

\[
\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} J_d Q & B \\ \mathcal{M} B^T Q & \mathcal{M} D + \mathcal{N} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}
\]

(1)
Time-discretization using energy-conserving methods can be used.
Time-discretization using energy-conserving methods can be used

Simulation of beam equations after initial condition: Energy

More information: June, 18th, ROMA Seminar by Said Aoues!
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1. Introduction
2. Fluid-structure system with piezo actuators
3. Discretization
4. Preliminary results
5. Limitations and further work
Numerical results show good agreement with experiments.

Natural frequencies (in Hz) - 25% filled tank

<table>
<thead>
<tr>
<th>Number of elements:</th>
<th>10</th>
<th>50</th>
<th>200</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slosh+bending</td>
<td>0.4318</td>
<td>0.4332</td>
<td>0.4332</td>
<td>0.46</td>
</tr>
<tr>
<td>Slosh+bending</td>
<td>1.1404</td>
<td>1.1436</td>
<td>1.1437</td>
<td>1.15</td>
</tr>
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<td>Slosh+bending</td>
<td>1.3690</td>
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<tr>
<td>Slosh+bending</td>
<td>2.0544</td>
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<td>2.5799</td>
<td>3.1637</td>
<td>3.1880</td>
<td>2.94</td>
</tr>
<tr>
<td>1st Torsion</td>
<td>8.4273</td>
<td>8.4268</td>
<td>8.4267</td>
<td>8.01</td>
</tr>
<tr>
<td>2nd bending</td>
<td>8.8955</td>
<td>8.7469</td>
<td>8.7515</td>
<td>9.61</td>
</tr>
</tbody>
</table>
Numerical results show good agreement with experiments.
Numerical results show good agreement with experiments

2nd mode
Numerical results show good agreement with experiments

3rd mode
We can perform (nonlinear) simulations
Some results of PH-based control

Damping assignment with state-observer:

- Graph showing speed (m/s) over time (s) with different damping parameters.
- Graph showing tension (Volts) over time (s) with different damping parameters.

Flavio Luiz Cardoso Ribeiro
Port-Hamiltonian Systems for Fluid and Structural

Limitations and further work
Some results of PH-based control

Damping assignment with state-observer:

![Graph showing damping assignment with state-observer](image)
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Nonlinear sloshing appears with high amplitudes

Harmonic excitation near 8.7 Hz (2nd bending mode)
Nonlinear sloshing appears with high amplitudes

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Nonlinear sloshing appears with high amplitudes

Harmonic excitation near 8.7 Hz (2nd bending mode)
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Harmonic excitation near 8.7 Hz (2nd bending mode)
Nonlinear sloshing appears with high amplitudes

Harmonic excitation near 8.7 Hz (2nd bending mode)
Saint-Venant sloshing model works only for small filling ratio

Bode plot of:

\[ G(s) = \frac{F(s)}{\ddot{W}(s)} \]

\[ = \frac{\text{fluid force on walls}}{\text{tank acceleration}} \]
Saint-Venant sloshing model works only for small filling ratio

Bode plot of:

\[ G(s) = \frac{F(s)}{\ddot{W}(s)} = \frac{\text{fluid force on walls}}{\text{tank acceleration}} \]
Saint-Venant sloshing model works only for small filling ratio

Bode plot of:

$$G(s) = \frac{F(s)}{\ddot{W}(s)} = \frac{\text{fluid force on walls}}{\text{tank acceleration}}$$
Summary

Port-Hamiltonian formulation provides a:

- modular,
- physically motivated (energy-based),
- multi-domain

framework for analyzing, simulating and controlling (complex) systems.

We used this formulation to:

- Model and simulate a fluid-structure problem;
- Control by damping injection;
What comes next?

Further work (short term):
- Model piezoelectric material;
- More accurate sloshing model;
- Control of the full system.

Further work (long term):
- Distributed fluid-structure problems;
- ...
- Energy-based control of aeroelastic systems?
Introductory bibliography - Bond graph and PHS

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