Port-Hamiltonian Systems for Fluid and Structural mechanics: Part II.

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Overview

Introduction

The port-Hamiltonian formalism brings together:

- Port-based modeling approach (bond-graph);
- Geometric mechanics (coordinate-free);
- Systems and control theory.

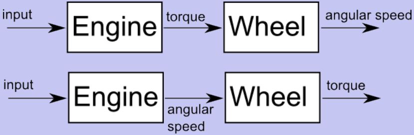
Bond graphs provide a unified framework for the modeling of physical systems

Henry Paynter (1959)

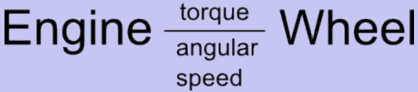
- Different domains (mechanical, electrical, hydraulic, thermal)
- Energy is the *lingua franca*;
- Complex systems are written as a composition of ideal components: energy-storage, energy-dissipation, energy-routing, etc.

Bond graph vs block diagram

Block diagram

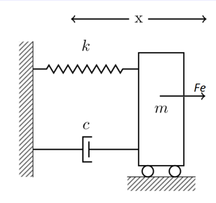


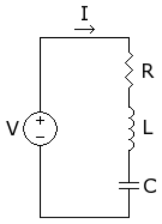
Bond Graph

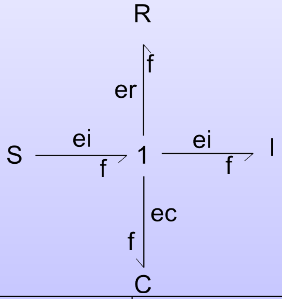


"Bonds" are energy-conserving and acausal!

Bond graphs provides a unified language for systems of different physical domains

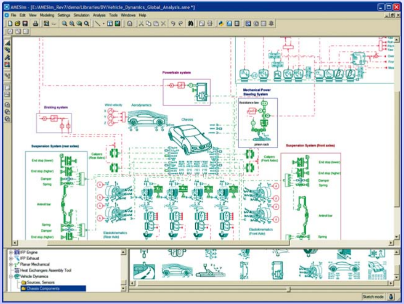






	Mass-spring-damper	RLC
Flow variables (f)	speed	current
Effort variables (e)	force	voltage
I storage (I)	mass (Inertia)	inductor
C storage (C)	spring	capacitor
Dissipation element (R)	damper	resistor

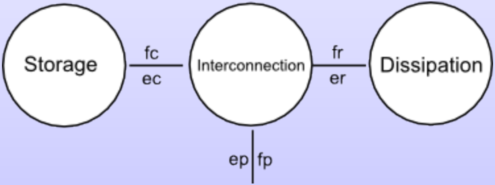
Bond graphs are used to model COMPLEX systems



Many commercial software based on bond-graphs:

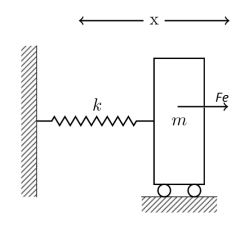
- AMEsim
- 20-sim (U. Twente)
- Simscape (Matlab/Simulink)
- Dymola (Catia / Dassault)

Port-Hamiltonian systems = bond graph elements + Hamiltonian



Storage function is the Hamiltonian (energy) $H(x)$.
Storage flow variable: $f_c = -\dot{x}$
Storage effort variable: $e_c = \frac{\partial H}{\partial x}(x)$
Energy flow: $\dot{H}(t) = \frac{\partial^T H}{\partial x}\dot{x} = -e_c^T f_c = e_p^T f_p + e_r^T f_r$

Simple example of port-Hamiltonian system



From Newton 2nd Law:

$$m\ddot{x} + kx = F_{ext},$$

System energy:

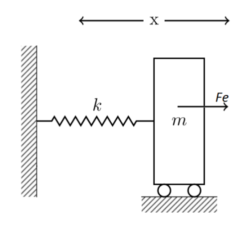
$$E = \frac{m\dot{x}^2}{2} + \frac{kx^2}{2}$$

By choosing $p = m\dot{x}$ then $H(p,x) = \frac{p^2}{2m} + \frac{kx^2}{2}$

$$\begin{bmatrix}\dot{p}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\frac{p}{m}\\ kx\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix} F_{ext},$$

$$\dot{x} = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\frac{p}{m}\\ kx\end{bmatrix}$$

Simple example of port-Hamiltonian system



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$$\underbrace{\begin{bmatrix}\dot{p}\\ \dot{x}\end{bmatrix}}_{-f_c} = \underbrace{\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}}_{J}\underbrace{\begin{bmatrix}\partial_p H\\ \partial_x H\end{bmatrix}}_{e_c} + \begin{bmatrix}1\\ 0\end{bmatrix}\underbrace{F_{ext}}_{e_p},$$

$$\underbrace{\dot{x}}_{f_p} = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\partial_p H\\ \partial_x H\end{bmatrix}$$

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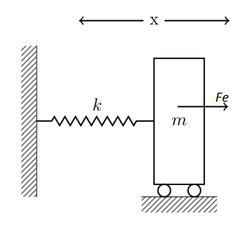

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$$\underbrace{\dot{x}}_{f_p} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \partial_p H \\ \partial_x H \end{bmatrix}$$

$\Longrightarrow \dot{H} = F_{ext}\dot{x}$

F_{ext} and $\dot{x}$ are the interconnection ports.

Typical mathematical representation of finite-dimensional port-Hamiltonian systems

$$\dot{x} = J\frac{\partial H}{\partial x} + Bu,$$
$$y = B^T\frac{\partial H}{\partial x}.$$

where:
$H(x)$: system Hamiltonian;
$x \in R^n$: energy variables;
$u \in R^m$: inputs;
$y \in R^m$: outputs;
J: interconnection matrix (skew-symmetric);

$$\Longrightarrow \dot{H} = y^T u$$

The interconnection of two (N) PHS is still a PHS

Individual systems

$$\dot{x}_1 = J_1 \frac{\partial H_1}{\partial x_1} + B_1 u_1,$$
$$y_1 = B_1^T \frac{\partial H_1}{\partial x_1},$$

$$\dot{x}_2 = J_2 \frac{\partial H_2}{\partial x_2} + B_2 u_2,$$
$$y_2 = B_2^T \frac{\partial H_2}{\partial x_2},$$

Interconnection

$$u_1 = y_2 + u_e,$$
$$u_2 = -y_1$$

$$\Longrightarrow$$

Coupled system

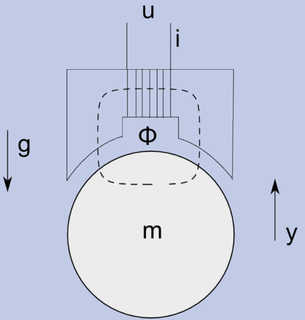
$$H(x_1, x_2) = H_1(x_1) + H_2(x_2)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} J_1 & B_1 B_2^T \\ -B_2 B_1^T & J_2 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u_e,$$
$$y_e = \begin{bmatrix} B_1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \end{bmatrix}$$

$$\frac{dH}{dt} = y_e^T u_e$$

PHS is a convenient way to model multi-physics systems

Ex.: Levitating iron ball in a magnetic field



$$\frac{dx}{dt} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \frac{\partial H}{\partial x} = i$$

$$H = \frac{x_1^2}{2L(x_2)} - mgx_2 + \frac{x_3^2}{2m},$$

$$\dot{H} = ui.$$

$x_1 = \phi$: total magnetic flux
$x_2 = y$: displacement of the ball
$x_3 = mv = p$: kinetic momentum

$\partial_{x_1} H$: current through the coil
$\partial_{x_2} H$: velocity of the ball
$\partial_{x_3} H$: electro-motive + gravity force

PHSs are convenient for (non-linear) control design

Typical port-Hamiltonian representation:

$$\dot{x} = J\frac{\partial H}{\partial x}(x) + Bu,$$
$$y = B^T\frac{\partial H}{\partial x}(x).$$

We've seen that: $\dot{H} = y^T u$

What happens if $u = -K(x)y$, with $K(x) > 0$?

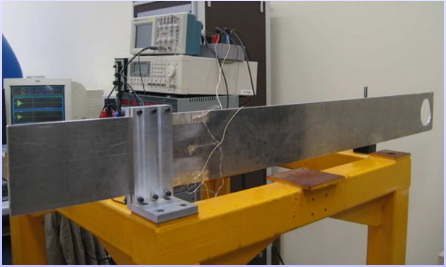
$$\dot{H} = -y^T K(x) y \leq 0,$$

If $H(x)$ is lower bounded, the controlled system is stable!

Overview

Fluid-structure excited by piezoelectric actuators

- Very flexible plate



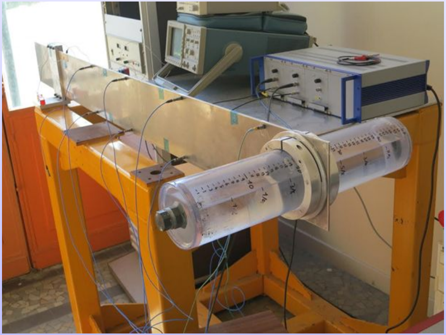
Fluid-structure excited by piezoelectric actuators

- Very flexible plate

$+$

- Partially filled tank

$=$ fluid/structure interactions



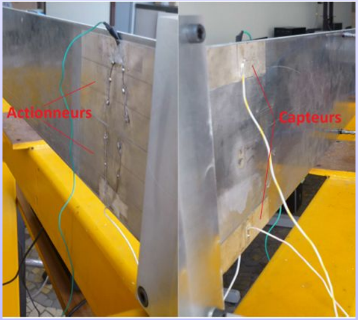
Fluid-structure excited by piezoelectric actuators

Proposition:

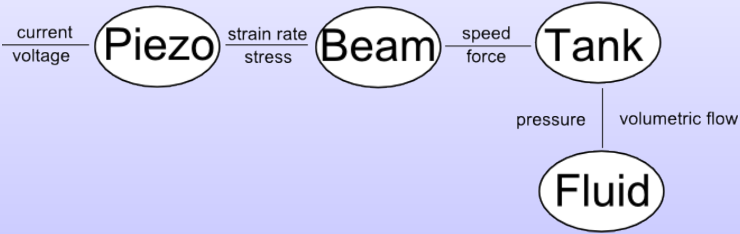
- Use piezoelectric patches as actuators (or sensors)
- Reduce vibrations and liquid sloshing by feedback

Problems:

- How to model the experiment?
- How to design a *robust* control law?



The problem can be decomposed into several subsystems that exchange energy



Some reasons for using PHS

- Multi-domain;
- Power-conserving;
- Object-oriented approach (modularity);

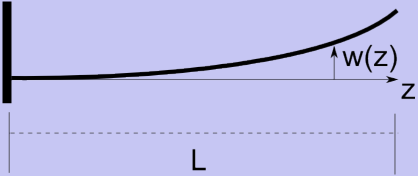
The beam can be written in PHS form (1)

Euler-Bernoulli

$$\mu \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial z}\left(EI \frac{\partial^2 w}{\partial z^2} \right) = 0,$$

boundary conditions:

- Fixed end (z = 0):
  $w(0, t) = 0$ and
  $\frac{\partial w}{\partial z}(0, t) = 0$
- Free end (z = L):
  $EI \frac{\partial^3 w}{\partial z^3}$ = tip force and
  $EI \frac{\partial^2 w}{\partial z^2}$ = tip moment



The beam can be written in PHS form (2)

Euler-Bernoulli

$$\mu \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial z}\left(EI \frac{\partial^2 w}{\partial z^2}\right) = 0,$$

Port-Hamiltonian version

Defining: $x_1 := \mu \frac{\partial w}{\partial t}$ and $x_2 := \frac{\partial^2 w}{\partial z^2}$:

$$\begin{aligned} \frac{\partial x_1}{\partial t} &= - \frac{\partial^2}{\partial z^2}\left(EI x_2\right), \\ \frac{\partial x_2}{\partial t} &= \frac{\partial^2}{\partial z^2}\left(\frac{x_1}{\mu}\right). \end{aligned}$$

Energy function: $H = \frac{1}{2}\int_{z=0}^{L} (\underbrace{\frac{x_1^2}{\mu}}_{\text{Kinetic energy}} + \underbrace{EI x_2^2}_{\text{elastic energy}}) dz$

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Energy function: $H = \frac{1}{2} \int_{z=0}^{L} (\underbrace{\frac{x_1^2}{\mu}}_{\text{Kinetic energy}} + \underbrace{EI x_2^2}_{\text{elastic energy}}) dz$

The beam can be written in PHS form (3)

Port-Hamiltonian version

$x_1 := \mu \frac{\partial w}{\partial t}$ and $x_2 := \frac{\partial^2 w}{\partial z^2}$

$$\underbrace{\begin{bmatrix} \frac{\partial x_1}{\partial t} \\ \frac{\partial x_2}{\partial t} \end{bmatrix}}_{-f_c} = \underbrace{\begin{bmatrix} 0 & -\frac{\partial^2}{\partial z^2} \\ \frac{\partial^2}{\partial z^2} & 0 \end{bmatrix}}_{\mathcal{J}} \underbrace{\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}}_{e_c}$$

$$e_1 = \frac{\delta H}{\delta x_1} = EI \frac{\partial^2 w}{\partial z}: \text{ moment}$$
$$e_2 = \frac{\delta H}{\delta x_2} = \frac{\partial w}{\partial t}: \text{ speed}$$
$$e_{\partial 1} = \frac{\partial}{\partial z} \frac{\delta H}{\delta x_1} = \frac{\partial}{\partial z}\left(EI \frac{\partial^2 w}{\partial z}\right): \text{ force}$$
$$e_{\partial 2} = \frac{\partial}{\partial z} \frac{\delta H}{\delta x_2} = \frac{\partial}{\partial t} \frac{\partial w}{\partial z}: \text{ angular speed}$$

The energy flow becomes:

$$\dot{H} = \int_{z=0}^{L} e_1 \dot{x_1} + e_2 \dot{x_2} dz = \int_{z=0}^{L} -e_1 \frac{\partial^2 e_2}{\partial z^2} + e_2 \frac{\partial^2 e_1}{\partial z^2} dz$$

$$= \int_{z=0}^{L} \frac{\partial e}{\partial z} \left(-e_1 \frac{\partial e_2}{\partial z} + e_2 \frac{\partial e_1}{\partial z}\right) dz = \left[- \underbrace{e_1}_{\text{moment}} \underbrace{e_{\partial 2}}_{\text{angular speed}} + \underbrace{e_2}_{\text{speed}} \underbrace{e_{\partial 1}}_{\text{force}} \right]_{z=0}^{L}$$

The fluid can be written in PHS form (1)

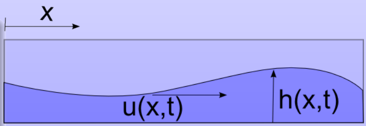
1D Saint-Venant

Hypothesis

- Incompressible fluid;
- Shallow water;
- Only hydrostatic pressure: $P = \rho g h$

Equations

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}(hu)$$
$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - g\frac{\partial}{\partial x}h$$

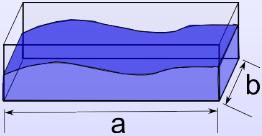


- u: fluid speed;
- h: fluid height;

The fluid can be written in PHS form (2)

$$\rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\frac{u^2}{2} = -g\frac{\partial h}{\partial x} \quad \text{(Navier-Stokes)},$$

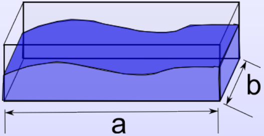
$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}(hu) \quad \text{(Mass conservation)}.$$



with energy given by:

$$H(u,h) = \frac{1}{2}\int_{x=-a/2}^{a/2} (\underbrace{\rho b u^2 h}_{\text{kinetic energy}} + \underbrace{\rho b g h^2}_{\text{potential energy}})dx.$$

The fluid can be written in PHS form (2)



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$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}(hu) = \qquad -\frac{\partial}{\partial x}\left(\frac{1}{\rho b}\frac{\delta H}{\delta h}\right)$$

$$\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\left(\frac{u^2}{2} + gh\right) = \qquad -\frac{\partial}{\partial x}\left(\frac{1}{\rho b}\frac{\delta H}{\delta h}\right)$$

The fluid can be written in PHS form (3)

Saint-Venant 1D: written in PH form

Defining: $\alpha_1 := \rho u$ and $\alpha_2 := bh$:

$$\frac{\partial}{\partial t}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \begin{bmatrix}0 & -\partial_x\\ -\partial_x & 0\end{bmatrix}\begin{bmatrix}\frac{\delta H}{\delta \alpha_1}\\ \frac{\delta H}{\delta \alpha_2}\end{bmatrix}$$

$$H(\alpha_1, \alpha_2) = \frac{1}{2}\int_{x=-a/2}^{a/2}\left(\frac{\alpha_1^2 \alpha_2}{\rho} + \rho g(\frac{\alpha_2^2}{b})\right) dx.$$

where:

$$e_1^F = \frac{\delta H}{\delta \alpha_1} = bhu = \text{volumetric flow} \quad , \quad e_2^F = \frac{\delta H}{\delta \alpha_2} = \rho\frac{u^2}{2} + \rho g h = \text{total pressure}$$

$$\frac{\partial H^F}{\partial t} = \underbrace{e_1^F(-a/2,t)}_{\text{vol. flow}}\underbrace{e_2^F(-a/2,t)}_{\text{pressure}} - \underbrace{e_1^F(a/2,t)}_{\text{vol. flow}}\underbrace{e_2^F(a/2,t)}_{\text{pressure}}$$

The tank can be written in PHS form

2nd Newton Law: $m_{RB}\ddot{w}_B(t) = F_{\text{ext}},$

Port-Hamiltonian equations

$$\frac{\partial}{\partial t}\begin{bmatrix} p \\ w_B \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \partial_p H^{RB} \\ \partial_{w_b} H^{RB} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F_{\text{ext}},$$

$$\dot{w}_B = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \partial_p H^{RB} \\ \partial_{w_b} H^{RB} \end{bmatrix}$$

where the Hamiltonian is equal to the kinetic energy: $H^{RB}(p) = \frac{1}{2}\frac{p^2}{m_{RB}}$, and its rate of change is given by:

$$\begin{aligned} \dot{H}^{RB} &= \dot{w}_B \dot{p} \\ &= \dot{w}_B F_{\text{ext}} . \end{aligned}$$

We can interconnect each component through their ports

Total Hamiltonian: $H = H^B + H^F + H^{RB}$

Kinematic constraint (equal speed):

$$\underbrace{\dot{w}_B}_{\text{tank}} = \underbrace{e_2^B(L,t)}_{\text{beam tip}} = \underbrace{e_1^F(-a/2,t)}_{\text{fluid boundary}}/S = \underbrace{e_1^F(a/2,t)}_{\text{fluid boundary a/2}}/S$$

Sum of forces equal to zero:

$$\mathcal{F} = \underbrace{-\frac{\partial}{\partial z} e_1^B(L,t)}_{\text{beam tip}} + \underbrace{F_{\text{ext}}}_{\text{tank}} \underbrace{-e_2^F(a/2,t)}_{\text{fluid boundary}} S + \underbrace{e_2^F(-a/2,t)}_{\text{fluid boundary}} S = 0$$

$$\implies \dot{H} = \dot{H}^B + \dot{H}^F + \dot{H}^{RB} = \dot{w}_B \mathcal{F} = 0$$

Overview

In order to simulate, we have to approximate infinite-dimensional equations

Spatial discretization methods:

- Mixed finite-elements: Golo 2003

$\Longrightarrow$ Port-Hamiltonian structure is conserved after discretization;
$\Longrightarrow$ No numerical dissipation;
$\Longrightarrow$ Boundary conditions (e_p) appears as interconnection ports

- Pseudo-spectral spatial sympletic reduction: Moulla 2012

Discretization methods leads to finite-dimensional PHS

Infinite-dimensional

$$\underbrace{\begin{bmatrix} \frac{\partial x_1}{\partial t} \\ \frac{\partial x_2}{\partial t} \end{bmatrix}}_{-f_c} = \underbrace{\begin{bmatrix} 0 & -\frac{\partial^2}{\partial z^2} \\ \frac{\partial^2}{\partial z^2} & 0 \end{bmatrix}}_{\mathcal{J}} \underbrace{\begin{bmatrix} \delta_{x_1} H \\ \delta_{x_2} H \end{bmatrix}}_{e_c}$$

$\Longrightarrow$

Finite-dimensional

$$\dot{\mathbf{x}} = J \frac{\partial H^d}{\partial \mathbf{x}} + Bu,$$

$$y = B^T \frac{\partial H^d}{\partial \mathbf{x}} + Du.$$

$$u = \begin{bmatrix} e_1^B(L) & e_{\partial 1}^B(L) & e_2^B(0) & e_{\partial 2}^B(0) \end{bmatrix}$$
$$y = \begin{bmatrix} -e_{\partial 2}^B(L) & e_2^B(L) & -e_{\partial 1}^B(0) & e_1^B(0) \end{bmatrix}$$

$$\dot{H} = \dot{H}^d = y^T u$$

Once we've discretized each subsystem, we find a set of DAE

Final discrete finite-dimensional system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{J_d}\frac{\partial H_d}{\partial \boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u}, \qquad \boldsymbol{u} = \begin{bmatrix} e^B_{1\partial}(L) & F^{RB}_{ext} & e^F_1(-a/2) & e^F_2(a/2) \end{bmatrix}^T$$

$$\boldsymbol{y^i} = (\boldsymbol{B})^T\frac{\partial H_d}{\partial \boldsymbol{x}} + \boldsymbol{D}\boldsymbol{u}, \qquad \boldsymbol{y} = \begin{bmatrix} e^B_2(L) & \dot{w}^{RB}_B & e^F_2(-a/2) & e^F_1(a/2) \end{bmatrix}^T$$

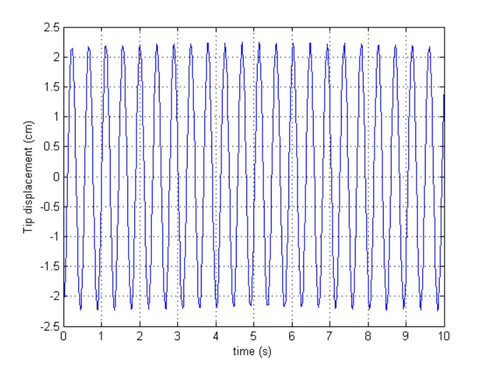
$$\mathcal{M}y + \mathcal{N}u = 0,$$

Coupled system: Differential Algebraic Equations

$$\underbrace{\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}}_{E} \frac{\partial}{\partial t}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{J_d Q} & \boldsymbol{B} \\ \mathcal{M}\boldsymbol{B}^T\boldsymbol{Q} & \mathcal{M}\boldsymbol{D} + \mathcal{N} \end{bmatrix}}_{A} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{bmatrix} \tag{1}$$

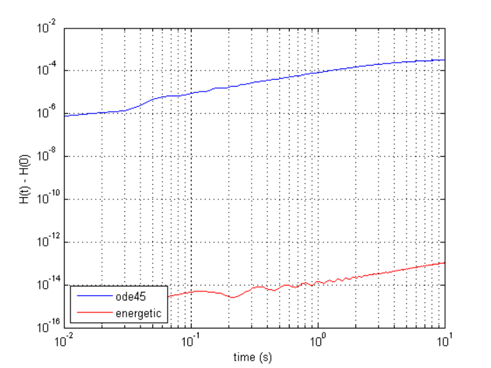
Time-discretization using energy-conserving methods can be used

Simulation of beam equations after initial condition



Time-discretization using energy-conserving methods can be used

Simulation of beam equations after initial condition: Energy



More information: June, 18th, ROMA Seminar by Said Aoues!

Overview

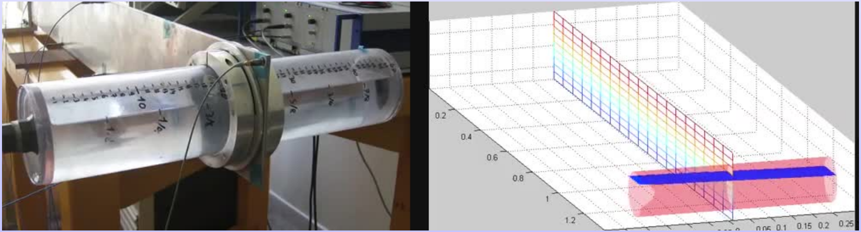
Numerical results show good agreement with experiments

Natural frequencies (in Hz) - 25% filled tank

Number of elements:	10	50	200	Experimental
Slosh+bending	0.4318	0.4332	0.4332	0.46
Slosh+bending	1.1404	1.1436	1.1437	1.15
Slosh+bending	1.3690	1.4174	1.4194	1.50
Slosh+bending	2.0544	2.2770	2.2860	2.38
Slosh+bending	2.5799	3.1637	3.1880	2.94
	...	...	...	...
1st Torsion	8.4273	8.4268	8.4267	8.01
2nd bending	8.8955	8.7469	8.7515	9.61

Numerical results show good agreement with experiments

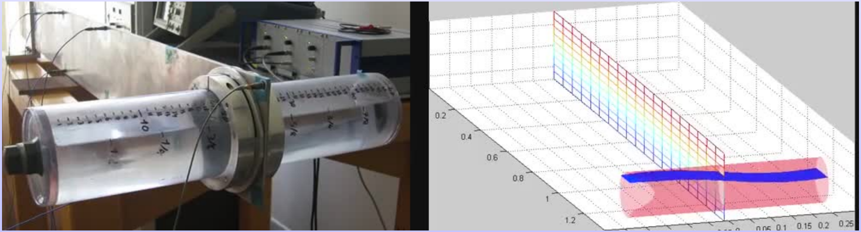
▷



1st mode

Numerical results show good agreement with experiments

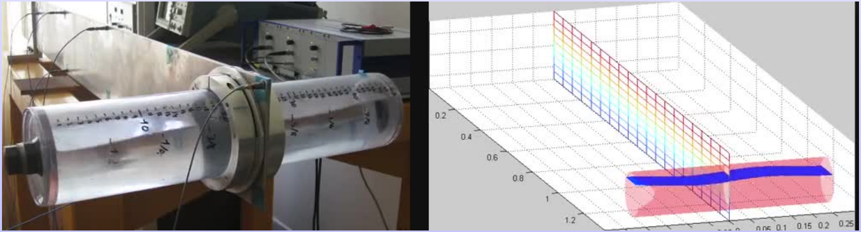
▷



2nd mode

Numerical results show good agreement with experiments

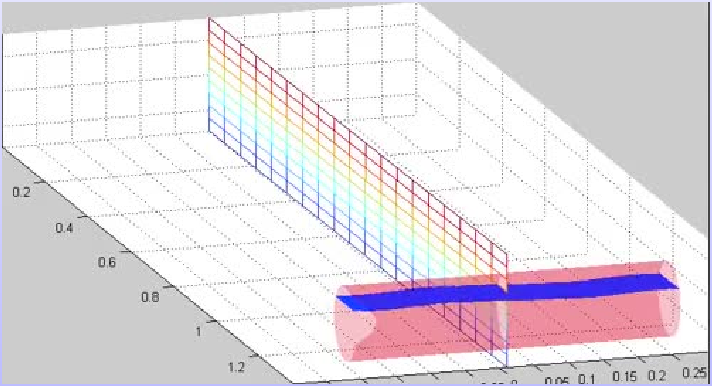
▷



3rd mode

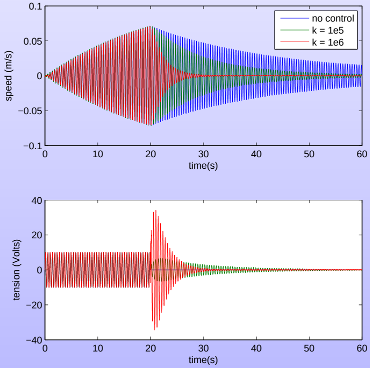
We can perform (nonlinear) simulations

▷



Some results of PH-based control

Damping assigment with state-observer:



Some results of PH-based control

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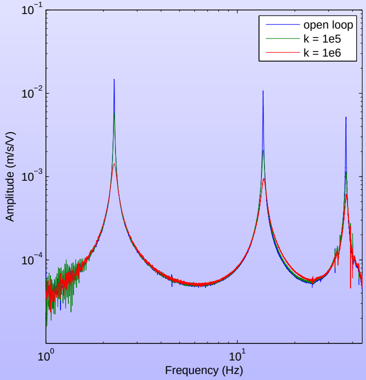
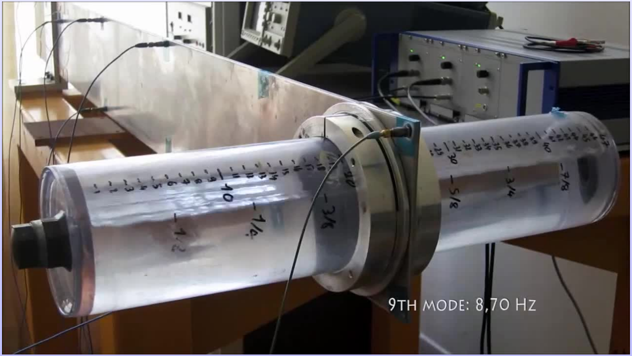


Overview

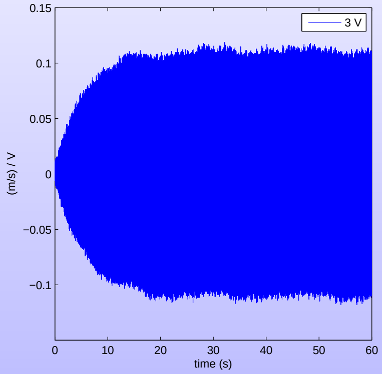
1. Introduction
2. Fluid-structure system with piezo actuators
3. Discretization
4. Preliminar results
5. **Limitations and further work**

Nonlinear sloshing appears with high amplitudes

▷ 

Harmonic excitation near 8.7 Hz (2nd bending mode)

Nonlinear sloshing appears with high amplitudes



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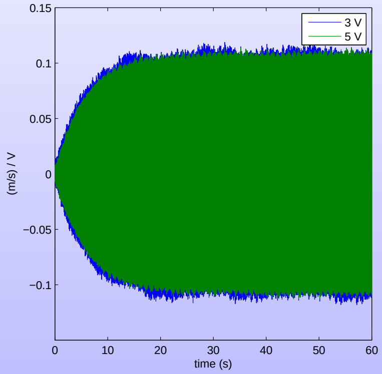

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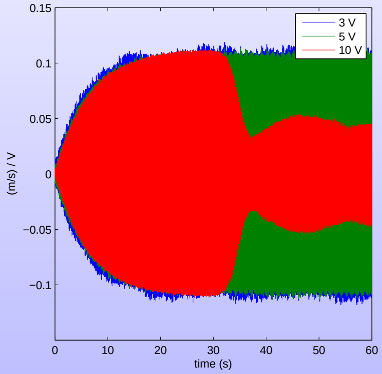

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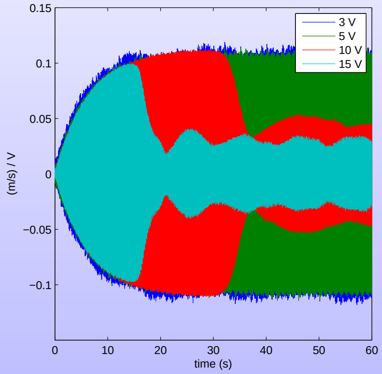
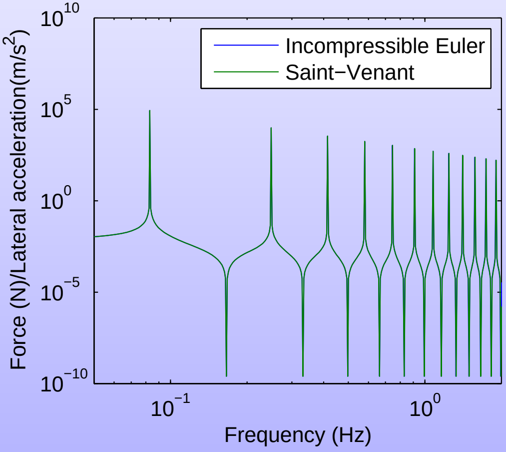
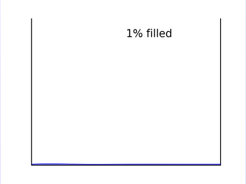


Harmonic excitation near 8.7 Hz (2nd bending mode)

Saint-Venant sloshing model works only for small filling ratio

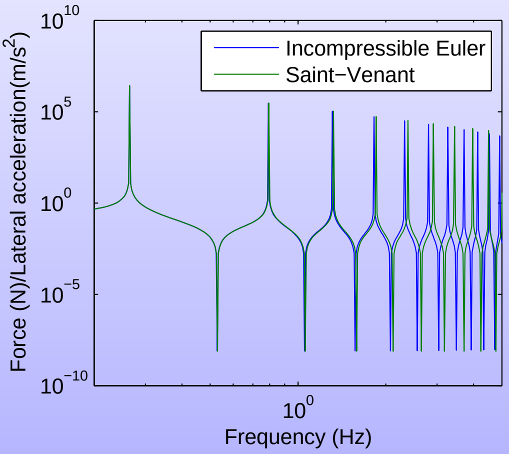


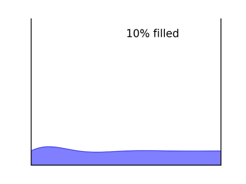


Bode plot of:

$$G(s) = \frac{F(s)}{\ddot{W}(s)} = \frac{\text{fluid force on walls}}{\text{tank acceleration}}$$

Saint-Venant sloshing model works only for small filling ratio





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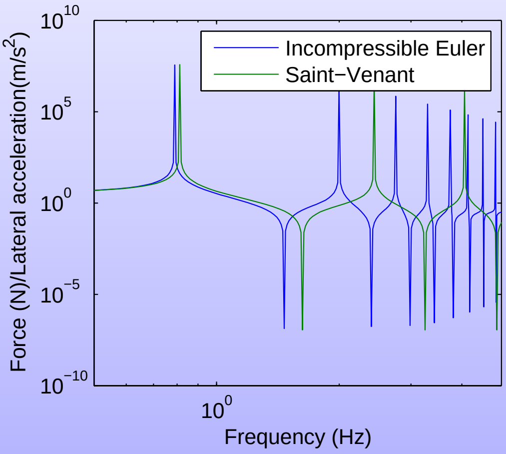

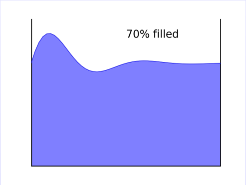

Bode plot of:

$$G(s) = \frac{F(s)}{\ddot{W}(s)} = \frac{\text{fluid force on walls}}{\text{tank acceleration}}$$

Summary

Port-Hamiltonian formulation provides a:

- modular,
- physically motivated (energy-based),
- multi-domain

framework for analyzing, simulating and controlling (complex) systems.

We used this formulation to:

- Model and simulate a fluid-structure problem;
- Control by damping injection;

What comes next?

Further work (short term):

- Model piezoelectric material;
- More accurate sloshing model;
- Control of the full system.

Further work (long term):

- Distributed fluid-structure problems;
- ...
- Energy-based control of aeroelastic systems?

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