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AEROFLEX: A TOOLBOX FOR STUDYING THE FLIGHT DYNAMICS OF HIGHLY FLEXIBLE AIRPLANES

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Abstract: *This work addresses a mathematical formulation to model highly flexible airplanes. A toolbox was developed and can be used to analyze how structural flexibility affects the airplane flight dynamics. A nonlinear beam model was applied to represent the structural dynamics, taking into account large displacements. For aerodynamic calculations, the strip theory was used including three modeling approaches: a quasi-steady, quasi-steady with apparent mass and full unsteady aerodynamics representations. Nonlinear simulations are performed and, through linearization of the equations of motion, dynamic stability is analyzed.*
Keywords: *aeroelasticity, flight dynamics, flexible airplanes, structural dynamics*

1. INTRODUCTION

Although all airplanes are flexible, rigid body assumption is very usual during the studies of flight dynamics. The effects of flexibility are usually taken into account by the discipline of aeroelasticity. This separation between aeroelasticity and rigid body flight dynamics used to be enough to describe these phenomena, but recent progress in aeronautical engineering with the advent of lighter structural materials has led to more flexible airplanes and urged the development of complete flight dynamics models including structural flexibility effects.

Waszak and Schmidt (1988) described dynamic equations of motions that include a linearized structural model. This approach can be used to study the influence of small structural deflections in the rigid body flight dynamics. Silvestre and Paglione (2008), Pogorzelski (2010) and Silva *et al.* (2010) used this formulation to study the flight dynamics and control of flexible airplanes. Among their hypothesis, these works neglect the inertial coupling between rigid body and flexible modes.

The recent development of High-Altitude Long-Endurance (HALE) airplanes increased even more the need for appropriate modeling of highly flexible aicrafts: since they have very high aspect ratio and low structural rigidity, their wings present large structural deflections as it can be seen in Figure 1.

Patil (1999) used a nonlinear beam model from Hodges (1990) to describe the flight dynamics of highly flexible airplanes. Brown (2003) modified the formulation, rewriting the equations in a strain-based form; he developed a framework for studying of wing warping as a means of achieving aeroelastic goals. Subsequently, Shearer (2006) improved the modeling, replacing numerical iterative calculations by closed form expressions. Su (2008) included absolute and relative nodal displacement constraints, allowing the study of Joined-Wing configurations.

AeroFlex is a toolbox that intends to implement the formulations of Brown (2003) and Shearer (2006), allowing the study of highly flexible airplanes flight dynamics. Among its main capabilities:

- Simulation and stability analysis of classic wing aeroelastic phenomena like: divergence, flutter, aileron reversals;
- Simulation and stability analysis of nonlinear wing aeroelastic phenomena, due to nonlinear geometry deflections;
- Simulation and stability analysis of a flexible aircraft in free-flight condition.

The main goal of this work is to present the implementation of AeroFlex. The equations of motion are presented in Section 2. Section 3 describes how the equations are solved. Section 4 presents the AeroFlex computational environment. Section 5 shows the results of several studies performed using the toolbox with the goal of validating it. These results are compared with test cases presented in the literature.

Figure 1: NASA Helios - Ref. Noll *et al.* (2004).

2. THEORETICAL FORMULATION

The equations of motion can be obtained from the principle of virtual work. The deduction is presented by Brown (2003), Shearer (2006), Su (2008) and Ribeiro (2011).

A three-dimensional structural model is decomposed in a bidimensional (cross-sectional) analysis. The results from the cross-section can be used to analyze a unidimensional beam. Each flexible structural member of the airplane is treated as a beam. These beams are split in several elements, each one can undergo deformations of extension, flexion and torsion. The deformations vector ϵ represents the deformation of each structural element. Linear and rotational speeds are represented by β.

The following equations of motion represent both the rigid body motion and the structural dynamics.

$$\begin{bmatrix} M_{FF} & M_{FB} \\ M_{BF} & M_{BB} \end{bmatrix} \begin{bmatrix} \ddot{\epsilon} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} C_{FF} & C_{FB} \\ C_{BF} & C_{BB} \end{bmatrix} \begin{bmatrix} \dot{\epsilon} \\ \beta \end{bmatrix} + \begin{bmatrix} K_{FF} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon \\ \vec{b} \end{bmatrix} = \begin{bmatrix} R_F \\ R_B \end{bmatrix} \tag{1}$$

where:

$$\begin{aligned} M_{FF}(\epsilon) &= J_{h\epsilon}^T M J_{h\epsilon} & M_{FB}(\epsilon) &= J_{h\epsilon}^T M J_{hb} \\ M_{MF}(\epsilon) &= J_{hb}^T M J_{h\epsilon} & M_{BB}(\epsilon) &= J_{hb}^T M J_{hb} + M_{RB} \\ C_{FF}(\epsilon, \dot{\epsilon}) &= J_{h\epsilon}^T M \dot{J}_{h\epsilon} + C & C_{FB}(\epsilon, \dot{\epsilon}, \beta) &= J_{h\epsilon}^T M H_{hb} + 2 J_{h\epsilon}^T M \dot{J}_{hb} + C_{RB} \\ C_{BF}(\epsilon, \dot{\epsilon}) &= J_{hb}^T M \dot{J}_{h\epsilon} & C_{BB}(\epsilon, \dot{\epsilon}, \beta) &= J_{hb}^T M H_{hb} + 2 J_{hb}^T M \dot{J}_{hb} \\ K_{FF} &= K \end{aligned} \tag{2}$$

$$\begin{bmatrix} R_F \\ R_B \end{bmatrix} = \begin{bmatrix} J_{p\epsilon}^T \\ J_{pb}^T \end{bmatrix} F^{pt} + \begin{bmatrix} J_{\theta\epsilon}^T \\ J_{\theta b}^T \end{bmatrix} M^{pt} + \begin{bmatrix} J_{p\epsilon}^T \\ J_{pb}^T \end{bmatrix} B^F F^{dist} + \begin{bmatrix} J_{\theta\epsilon}^T \\ J_{\theta b}^T \end{bmatrix} B^M M^{dist} + \begin{bmatrix} J_{h\epsilon}^T \\ J_{hb}^T \end{bmatrix} N\vec{g} + \begin{bmatrix} 0 \\ R_{RB}^{ext} \end{bmatrix} \tag{3}$$

In Eq. 1, M_{ij} represents the mass matrix; $C_{i,j}$ represents the damping matrix; K_{FF} represents the structural rigidity matrix. It's possible to see that rigid body states (represented by β) are inertially coupled with structural states (ϵ), since the mass matrix is not diagonal.

R_F and R_B represents the generalized forces that are applied in the airplane. They are obtained from the aerodynamic, gravitational and propulsive forces applied to each structural node. Strip theory is applied, so that aerodynamic forces and moments are calculed using bidimensional models in each node. The aerodynamic models are presented in Section 2.1.

The Jacobian matrices $J_{h\epsilon}$ and $J_{\theta\epsilon}$ represent the relationship between structural deformations (ϵ) and nodal displacements and rotations. J_{hb} and $J_{\theta b}$ represent the relationship between rigid body degrees of freedom and nodal displacements and rotations. The Jacobian matrices are nonlinear functions of ϵ. They can be obtained either numerically (through numerical linearization), or through analytical expressions as presented by Shearer (2006). This work uses the latter.

In addition, M is the flexible structure mass matrix. It depends only on inertias and masses of the structural elements (it is not dependent on strain, differently from the M_{ij} matrix). K is the structural rigidity matrix and C is the structural damping matrix. In AeroFlex, we usually uses a linear relationship between C and K, given by:

$$C = cK \tag{4}$$

where c is the damping ratio.

Euler angles are used to describe the airplane attitude (ϕ,θ and ψ, which are used to describe the bank, pitch and heading angles). Stevens and Lewis (2003) show that the time rate derivative of Euler angles are related with angular speeds (P,Q,R) by the following expressions:

$$\dot{\theta} = Q\cos\phi - R\sin\phi \tag{5}$$

$$\dot{\phi} = P + \tan\theta\left(Q\sin\phi + R\cos\phi\right) \tag{6}$$

$$\dot{\psi} = \frac{\left(Q\sin\phi + R\cos\phi\right)}{\cos\theta} \tag{7}$$

Stevens and Lewis (2003) also presents the relationship between speeds in the Body Frame (given by U,V,W) and Inertial Frame (given by $\dot{H}$, $\dot{x}$ and $\dot{y}$):

$$\dot{H} = U\sin\theta - V\sin\phi\cos\theta - W\cos\phi\cos\theta \tag{8}$$

$$\dot{x} = U\cos\theta\cos\psi + V(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + W(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \tag{9}$$

$$\dot{y} = U\cos\theta\sin\psi + V(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + W(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \tag{10}$$

2.1 Aerodynamic Models

The aerodynamic model is included through bidimensional forces and moments distributed along the beams. In this work, strip theory is used: aerodynamic forces and moments are calculated in each node considering an independent two-dimensional aerodynamic model.

Once aerodynamic forces (drag and lift) and moment are calculated; forces and moments are transformed from the local aerodynamic frame to body frame. The force vectors are then arranged in a 9N vector (where N is to number of elements). These vectors, of distributed forces and moments, are then applied to Eq. 3.

Three different models were used to calculate aerodynamic forces. The first consists in a quasi-steady model, which takes into account only the circulatory part of lift force, neglecting the wake effects. The second model includes the apparent mass terms. Finally, the third model is an unsteady aerodynamic model proposed by Peters *et al.* (2007), which includes states to represent the aerodynamic lag due the wake.

2.1.1 Quasi-steady

The following equations can be used to calculate the aerodynamic lift and moment around the elastic center for a flat plate (Ref. Haddadpour and Firouz-Abadi (2006)):

$$L = 2\pi\rho b U^2\left[\frac{\dot{h}}{U} + b(0.5-a)\frac{\dot{\alpha}}{U} + \alpha\right] \tag{11}$$

$$M_{ea} = b(0.5+a)L - \frac{\pi\rho U b^3}{2}\dot{\alpha} \tag{12}$$

In these equations, ρ is the air density; U is the airspeed; α is the local angle of attack; b is the airfoil semichord; and a is the distance between elastic axis and the half chord (normalized by the semichord b). It is possible to rewrite these equations as a function of relative speeds written in the zero-lift coordinate system[1].

$$L = 2\pi\rho b\dot{y}^2\left[\frac{-\dot{z}}{\dot{y}} + b(0.5-a)\frac{\dot{\alpha}}{\dot{y}}\right] \tag{13}$$

$$M_{ea} = b(0.5+a)L - \frac{\pi\rho\dot{y}b^3}{2}\dot{\alpha} \tag{14}$$

2.1.2 Quasi-steady with apparent mass

The following equations include the apparent mass effect (Ref. Haddadpour and Firouz-Abadi (2006)):

$$L = \pi\rho b^2\left[\ddot{h} - ba\ddot{\alpha} + U\dot{\alpha}\right] + 2\pi\rho b U\left[\dot{h} + b(0.5-a)\dot{\alpha} + U\alpha\right] \tag{15}$$

$$M_{ea} = b(0.5+a)L - \pi\rho b^3\left[0.5\ddot{h} + b(0.125 - 0.5a^2)\ddot{\alpha} + U\dot{\alpha}\right] \tag{16}$$

Again, it is possible to rewrite the equations as a function of variables written in the zero-lift coordinate system.

$$L = \pi\rho b^2(-\ddot{z} + \dot{y}\dot{\alpha} - d\ddot{\alpha}) + 2\pi\rho b\dot{y}^2\left[-\frac{\dot{z}}{\dot{y}} + \left(\frac{1}{2}b - d\right)\frac{\dot{\alpha}}{\dot{y}}\right] \tag{17}$$

$$M_{ea} = b(0.5+a)L - \pi\rho b^3\left[-0.5\ddot{z} + b(0.125 - 0.5a^2)\ddot{\alpha} + \dot{y}\dot{\alpha}\right] \tag{18}$$

[1] In the zero-lift coordinate system, y axis is parallel to the airfoil's zero-lift axis. The z axis is perpendicular, pointing upwards. The speeds in this coordinate system are represented here by $\dot{z}$ and $\dot{y}$.

2.1.3 Unsteady

An unsteady aerodynamic model based on Peters *et al.* (1995) is applied. Expressions for lift and drag are presented by Shearer (2006):

$$L = \pi\rho b^2(-\ddot{z} + \dot{y}\dot{\alpha} - d\ddot{\alpha}) + 2\pi\rho b\dot{y}^2\left[-\frac{\dot{z}}{\dot{y}} + \left(\frac{1}{2}b - d\right)\frac{\dot{\alpha}}{\dot{y}} - \frac{\lambda_0}{\dot{y}}\right] \tag{19}$$

$$M_{ea} = Ld + 2\pi\rho b^2\left(-\frac{1}{2}\dot{y}\dot{z} - \frac{1}{2}d\dot{y}\dot{\alpha} - \frac{1}{2}\dot{y}\lambda_0 - \frac{1}{16}b^2\ddot{\alpha}\right) \tag{20}$$

where λ_0 consists in:

$$\lambda_0 \approx \frac{1}{2}\sum_{n=1}^{N} b_n\lambda_n \tag{21}$$

where b_n can be obtained from the following expression (Ref. Peters *et al.* (1995)):

$$b_n = (-1)^{n-1}\frac{(N_A + n - 1)!}{(N_A - n - 1)!}\frac{1}{(n!)^2} \qquad 1 < n < N_A - 1$$
$$b_{N_A} = (-1)^{N_A+1} \tag{22}$$

The lag states λ_n can be obtained from the following system of differential equations:

$$\dot{\lambda} = E_1\lambda + E_2\ddot{z} + E_3\ddot{\alpha} + E_4\dot{\alpha} \tag{23}$$

On above axpressions, N_A is the number of aerodynamic lag states.λ_n; E_1, E_2, E_3 e E_4 are matrices presented in Ref. Balvedi (2010).

2.1.4 Drag

In the previous modelling approaches, the drag is calculated using a constant airfoil drag coefficient (C_{d0}):

$$D = \frac{1}{2}\rho\dot{y}^2 C_{d0} \tag{24}$$

2.1.5 Including trailing edge flap deflections

Trailing edge deflection is implemented by adding incremental values to the airfoil aerodynamic forces and moments:

$$L' = L + L^\delta \tag{25}$$
$$M' = M + M^\delta \tag{26}$$

where:

$$L^\delta \rho b\dot{y}^2 C_{L,\delta}\delta_u \tag{27}$$
$$M^\delta = \rho b^2 C_{m,\delta}\delta_u \tag{28}$$

$C_{L,\delta}$ and $C_{m,\delta}$ can be obtained through experimental data or airfoil analysis softwares. δ_u is the airfoil flap deflection.

2.2 Control inputs

Two types of control inputs are used in this modelation:

- Flap deflections δ_i (as presented in the previous section);
- Engine throttle π_i

Propulsion forces are modeled as point forces attached to a structural node.

3. SOLVING EQUATIONS, LINEARIZATION AND STABILITY

The elastic equations of motion (Eq. 1), the unsteady aerodynamic equations (Eq. 23) and the kinematics equations (Eqs. 5, 6, 7, 8, 9 and 10) represent all the needed expressions to describe the flight dynamics of the flexible airplanes. These are $4N$ second order differential equations to describe the structural dynamics, $3NN_A$ first order differential equations to describe the lag aerodynamic states and 12 first order equations to describe the rigid body motion [2].

Following, the methodologies used to find the equilibrium condition, integrate and linearize the equations of motion are presented.

3.1 Calculation of equilibrium

The determination of equilibrium condition in the case of the full airplane is done by the following iterative procedure:

1. Consider $\epsilon = 0$;
2. Calculate the rigid body equilibrium ($\dot{\beta} = 0$)[3];
3. Calculate the structural equilibrium ($\ddot{\epsilon} = 0$)[4];
4. Return to item 2 until both conditions are valid ($\dot{\beta} = 0$ e $\ddot{\epsilon} = 0$).

In the case of a straight level flight, for example, a specific flight condition is given (altitude, speed and angle of trajectory). Step 1 finds the engine throttle π, the elevator angle δ and the pitch angle θ. Step 2 finds the structural deformations ϵ. Each step is found by using numerical methods.

3.2 Integration of the nonlinear equations of motion

We have a large system of differential equations (Eqs. 1, 23, 5, 6, 7, 8, 9 and 10). Eq. 1 is a second order system of equations. To solve these equations we could try to convert this system in a first order system. Unfurtunately, this is not usually possible, since aerodynamic force expressions are nonlinear functions of the states' time rate. So two options are avaiable:

- Neglect the derivative terms from aerodynamic expressions and transform the system of equations into a first order system. This lead to a system of equations that can be integrated using classic explict methods (like Runge-Kutta methods);
- Integrate the second order system of equations using an implicit method.

The second option, though usually slower, is obviously the most precise.

3.3 Linearization and stability

It is possible to linearize the equations of motion with the goal of studying the stability of the flexible airplane. The system of equations can be represented in the following form:

$$f(\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \lambda, \dot{\lambda}, \beta, \dot{\beta}, \vec{k}, \dot{\vec{k}}, \delta_{u,i}, \pi_i) = 0 \tag{29}$$

where f is a nonlinear function which dimension is equal to the total number of system's states. $\vec{k}$ is the vector of kinematics variables:

$$\vec{k} = \begin{bmatrix} \phi & \theta & \psi & H \end{bmatrix} \tag{30}$$

It is possible to reduce it to a first order system (making $X = \dot{\epsilon}$) and linearize it around a equilibrium point:

$$M \begin{bmatrix} \dot{\tilde{X}} \\ \dot{\tilde{\epsilon}} \\ \dot{\lambda} \\ \dot{\tilde{k}} \end{bmatrix} = A \begin{bmatrix} \tilde{X} \\ \tilde{\epsilon} \\ \lambda \\ \tilde{k} \end{bmatrix} + B \begin{bmatrix} \tilde{\delta}_{u,i} & \tilde{\pi}_i \end{bmatrix} \tag{31}$$

The linearization is done numerically. By analysing the eigenvalues of $M^{-1}A$, we can verify if the system is stable. Choosing subsets of matrices M and A, it is possible to decouple the rigid body and structural dynamics. This allows, in a single process of linearization, determine the stability characteristics and autonomous response of the following systems:

[2] Where N is the total number of structural elements; N_A is the number of lag states in each node

[3] See that in the equilibrium: $M_{BB}\dot{\beta} = R_B$. So: $\dot{\beta} = 0$ is equivalent to $R_B = 0$. Where R_B is the sum of external forces, which is also a function of ϵ.

[4] We calculate ϵ so that $K_{FF}\epsilon = R_F$. Remember that R_F is a function of ϵ.

- Rigid body;
- Cantilevered wing;
- Flexible airplane in free flight.

To determine the instability speed (flutter, divergence or other), the following procedure is applied: the airplane speed is increased; for each speed, a new equilibrium condition is obtained; the system is linearized; the largest real part of the eigenvalues of $M^{-1}A$ is taken. Once one of the eigenvalues has a positive real part, the system is unstable. The imaginary part of this eigenvalue gives the frequency associated with the unstable aeroelastic mode.

4. CODE IMPLEMENTATION

AeroFlex is intended to perform the following tasks:

- Implement the strain-based geometrically nonlinear beam structural dynamics model proposed by Brown (2003) and the improvements proposed by Shearer (2006);
- Allow the study of airplanes with the following characteristics:

  - Rigid fuselage and flexible members (wings, horizontal and vertical tail);

  - Rigid concentrated mass/inertia elements attached to the flexible structure's nodes (to represent engines or fuel tanks for example);

  - Propulsive forces attached to the flexible structure's nodes.
- Determine the equilibrium point, considering the structural deformations;
- Linearize the equations of motion, allowing the dynamic stability study;
- Simulate the linear and nonlinear dynamics, using several numerical integration methods.

The code is intended to be very general, allowing the user to define new configurations easily. The authors chose to write the AeroFlex code using the Matlab ®[5].

4.1 Initializing the airplane data

AeroFlex was developed using object-oriented programming. Several classes were defined, in order to initialize and update the various data types that should be handled by the program. The four most important classes of this tool are the following: *node*, *element*, *engine*, *airplane*.

To initialize the airplane modelling, the user needs the following information: masses and inertias per unit of lenght at each structural node; rigidity and damping matrices of each element; lengh of each element and relative orientation beetwen one element and the next one. These data are obtained from the airplane geometry and from a cross-sectional analysis software. Additionaly, the user needs the aerodynamic data of each node (zero-lift angle of attack, C_{l_α}, C_{m_0}, number of lag-states, etc.).

Once all airplane properties are known, the following procedure should be done by the user:

1. Create *node* objects. Each object is a structural node and needs to be initialized with the mass and inertia data. These objects also have the information about the aerodynamic model (since the aerodynamic calculations are done in each node);
2. Create vectors of *element* objects. Each vector is a flexible member. Each unit of this vector is an element and it is associated with three *node* objects. These objects includes the rigidity and damping properties, associated with each element. The user can create how many flexible members are needed to describe the airplane;
3. Create *engine* objects. Each object of this class is an engine. To initialize this object, the user gives informations about the engine's position and about the propulsive model.
4. Create one *airplane* object. This object covers all the airplane data. Their input arguments are: vectors of *element* objects (members) and *engine* objects. In addition, the user can start this object with rigid fuselage data, if it exists.

Following this procedure, the user will have an object of the *airplane* class, which includes all the structural, aerodynamical and propulsive data of the airplane. This object allows the use of methods for calculating equilibrium; linearization; simulation and others. These methods are presented in the next items.

[5]Matlab 2010a, The MathWorks, Natick, MA

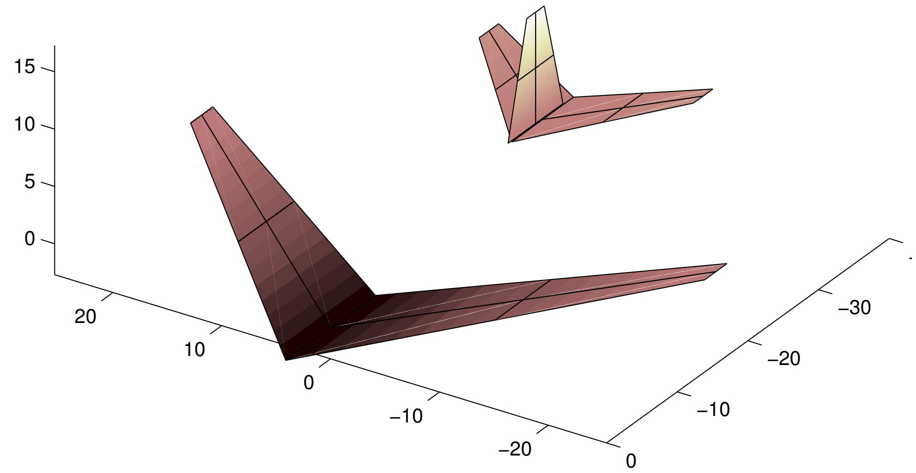

Figure 2: Example of airplane modeled in AeroFlex.

4.2 Equilibrium methods

One of the *airplane* class functions is the *trimairplane*. This function's goal is to calculate the structural and rigid body equilibrium given a straight flight condition (altitude and speed). The deformations vector ϵ, the elevator angle, the pitch angle and the throttle are the outputs for this function.

The methodology to find equilibrium is described in Section 3.1. AeroFlex uses FSOLVE Matlab function to solve the nonlinear equations.

4.3 Linearization method

The function *linearizeairplane* has the following input arguments: an *airplane* object and equilibrium conditions around which the linearized matrices should be calculated. The following outputs are presented by this function: A and B matrices of the full linearized system; in addition, matrices $A_{aeroelast}$ e A_{body} are the linearized system neglecting the rigid body and flexible degrees of freedom, respectively. Analyzing the eigenvalues of each of these matrices, it is possible to study the system stability.

Linearization is performed numerically as presented in Section 3.3.

4.4 Nonlinear simulation method

The function *simulate* is also a method of *airplane* class. It is intended to simulate the nonlinear dynamics. Its input arguments are: the *airplane* object; initial conditions; function handles to describe the engine throttle and elevator inputs as a function of time; integration method (implicit or explicit).

To integrate the equations of motion, the ODE15i and ODE15s Matlab functions are used (implicit and explicit methodologies, respectively).

4.5 Graphical outputs methods

The outputs of the simulation routines are vectors of the system's states for each instant of time (strain ϵ, linear and angular speeds $\vec{\beta}$, position and orientation of the body frame $\vec{k}$ and lag states λ). A function called *airplanemovie* was created allowing the presentation of a video with the airplane deflections along the time from the simulation results. The input arguments for it are: the *airplane* object; a time vector; a strain vector for each instant of time.

Additionally, the function *plotairplane3d* presents a 3D figure of object *airplane*. Figure 2 shows an example of this graphical output.

5. RESULTS

In order to validate the toolbox, several results were obtained and compared with literature. AeroFlex was used to solve structural problems; aeroelastic problems; and to make comparisons with rigid body flight dynamics. More results can be found in Ribeiro (2011).

5.1 Structural Problems

A simple cantivelered beam was modeled, as proposed by Ref. Brown (2003). Concentrated forces and moments were applied, as presented in Figure 3. Results show good agreement between this tool and the literature results, as shown in Figure 4.

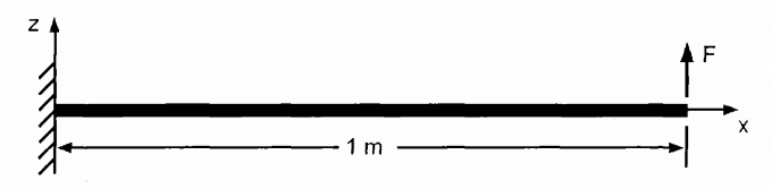

Figure 3: Cantilevered beam with a force applied - Ref. Brown (2003).

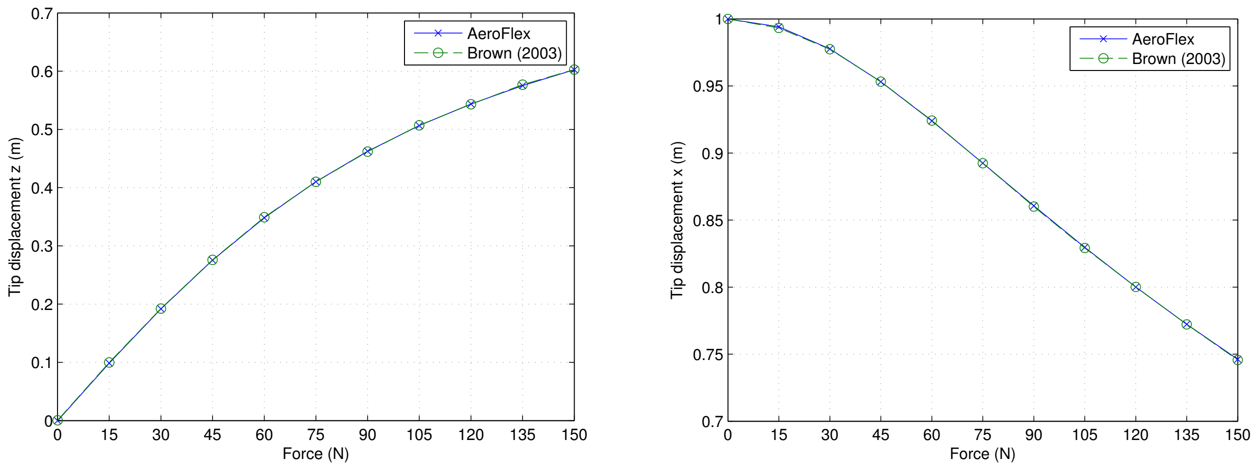

Figure 4: Deflection of tip as a result of a force.

5.2 Aeroelasticity

Two test cases presented in the literature were analyzed, with the goal of finding the flutter speed and frequency of cantilevered wings. In order to get these results, it is necessary to find the equilibrium condition for several different speeds, linearize the equations of motion and get the eigenvalues of the state matrix. Once at least one of the eigenvalues has a positive real part, the system is unstable. We can get the frequency of the unstable modes from the imaginary part of this eigenvalue (if it is oscilatory).

The first test case results are for the Goland wing (Ref. Goland (1945)) and can be seen in Table 1. After that, we show the results for a highly flexible wing, as proposed by Patil (1999). Due to its highly flexible nature, this wing shows an interesting result: if we study instability around an undeformed shape, very different results than those of a deformed shape are obtained (Fig. 5). Results are shown in Tables 2 and 3.

Table 1: Flutter speed and frequency for the Goland Wing.

Altitude	Results			
	AeroFlex		Ref.Brown (2003)	
	V (ft/s)	f (rad/s)	V (ft/s)	f (rad/s)
0 ft	451	71.2	447	69.7
20×10^3 ft	581	69.7	574	68.1

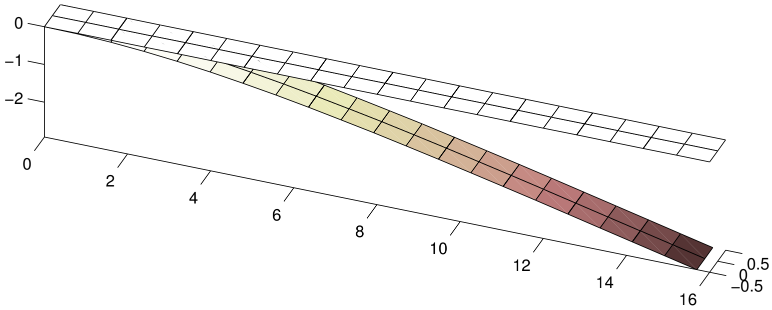

Figure 5: Highly Flexible Wing.

Table 2: Flutter speed and frequency for the highly flexible wing - undeformed wing.

	AeroFlex	Ref. Patil (1999)
Speed (m/s)	32.6	32.2
Frequency (rad/s)	22.6	22.6

Table 3: Flutter speed and frequency for the highly flexible wing - deformed wing.

	AeroFlex	Ref. Su (2008)
Speed (m/s)	23.4	23.2
Frequency (rad/s)	12.2	10.3

5.3 Flight Dynamics

In order to check if the flight dynamics simulated by AeroFlex agrees with a rigid body classical model (as of Ref. Stevens and Lewis (2003)), we've modeled a flying wing (Figure 6). Results for a doublet input in the elevator are presented in Fig. 7. For the very rigid airplane (K=1000), the results of AeroFlex and the rigid body dynamics are very similar. On the other hand, for a highly flexible airplane (K=1), we can see the coupling between structural and flight dynamics responses.

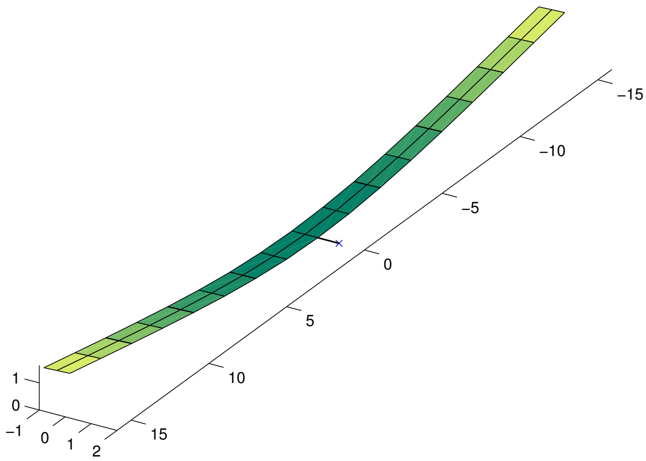

Figure 6: Flying wing.

6. CONCLUSIONS

This work presented the implementation of AeroFlex, a computational tool that allows the study of highly flexible airplane flight dynamics. This tool was used to study several test cases, from static structural problems to flight dynamics of flexible vehicles. Results obtained are very similar from those of the literature.

The methodology used is more suitable for studying airplanes with high aspect ration lifting surfaces, since it uses strip theory for aerodynamics and beam theory for structural dynamics. Ir order to study low aspect ratio wings, a threedimensional aerodynamic model would be necessary.

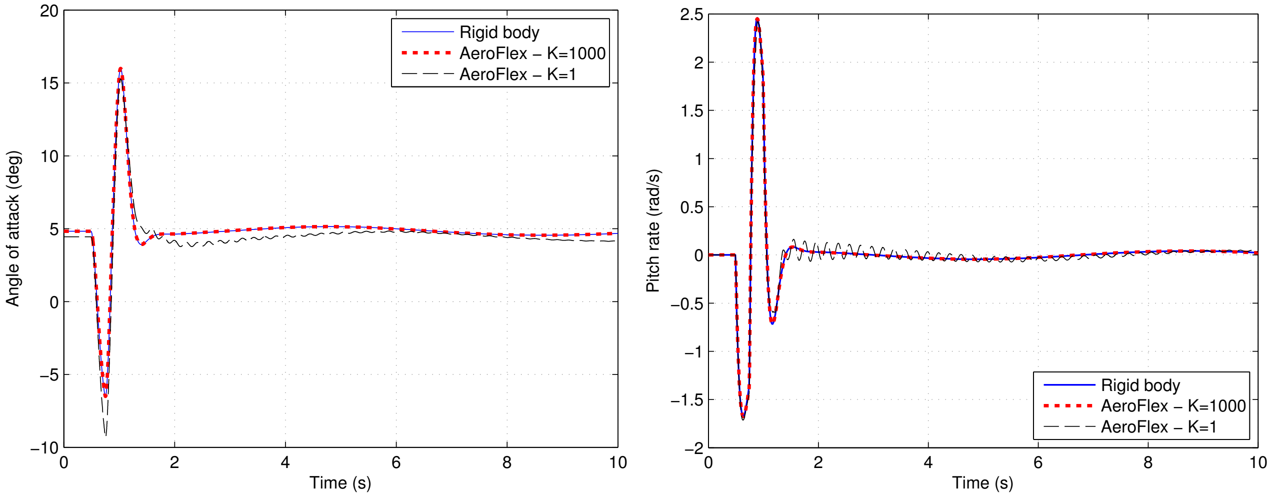

Figure 7: Simulated response for a doublet input in the elevator.

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