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STABILITY ANALYSIS OF A HIGHLY FLEXIBLE AIRPLANE

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Abstract:*This work addresses the study of a highly flexible flying wing. Due to the large structural deflections, the flight dynamics of this vehicle is strongly affected by the flexible behavior.*
A mathematical formulation that couples rigid body flight dynamics with a geometrically nonlinear beam model is applied. Aerodynamic forces and moments are calculated through unsteady strip theory. The equations of motion are linearized around equilibrium, allowing the study of dynamic stability.
Results show that, due to coupling between rigid body and aeroelastic dynamic modes, the instability can occur at speeds and frequencies much lower than usual flutter. For these airplanes, the usual analyses of aeroelasticity and flight dynamics as independent problems can produce mislead results.
Keywords: *aeroelasticity, flight dynamics, flexibility, flutter, flying wing*

1. INTRODUCTION

The coupling between aerodynamics and structural dynamics can lead to instability. Divergence and flutter are known since the early aviation history. (Weisshar, 1995)

In order to find the flutter speed, the structural dynamics model is usually considered as linear. This model is then coupled with an unsteady aerodynamic model. Through the analysis of the linearized system, it is possible to determine in which speed the system wiill be stable. (Bisplinghoff, 1996)

Patil (1999) used a nonlinear beam model from Hodges (1990) to analyze the flutter speed of a highly flexible cantilevered wing. He showed that in order to have more precise results than usual aeroelastic analysis, a nonlinear structural model is necessary.

Su and Cesnik (2010) used a modelling approach that couples the flight dynamics and aeroelasticity. This model was used to study the stability of a highly flexible airplane. Structural nodes' frequencies are low in this case. For this reason, the frequencies associated with the rigid body and structural modes are similar. This can lead to a low-frequency instability, associated with the coupling of these movements. This unstable phenomenon is usually called as "free-flight flutter".

This work uses the methodologies of Brown (2003), Shearer (2006) and Su (2008) to describe the flight dynamics of elastic airplanes. This formulation implements the geometrically nonlinear beam formulation and includes the inertial coupling between flight dynamics and flexible structure.

The main goal of this work is to analyze the stability of flexible airplanes and the "free-flight flutter". The nonlinear differential equations of elastic aircrafts flight dynamics are presented in Section 2. Section 3 describes the process of linearization and stability analysis. Section 4 describes the airplane studied in this paper. Finally, Section 5 shows the results of the stability analysis.

2. EQUATIONS OF MOTION

The equations of motion can be obtained from the principle of virtual work. The deduction is presented by Brown (2003), Shearer (2006), Su (2008) and Ribeiro (2011). The following set of differential equations, representing the full flexible airplane, are obtained:

$$\begin{bmatrix} M_{FF} & M_{FB} \\ M_{BF} & M_{BB} \end{bmatrix} \begin{bmatrix} \ddot{\epsilon} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} C_{FF} & C_{FB} \\ C_{BF} & C_{BB} \end{bmatrix} \begin{bmatrix} \dot{\epsilon} \\ \beta \end{bmatrix} + \begin{bmatrix} K_{FF} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon \\ \vec{b} \end{bmatrix} = \begin{bmatrix} R_F \\ R_B \end{bmatrix} \quad (1)$$

$$\dot{\lambda} = E_1\lambda + E_2\ddot{z} + E_3\ddot{\alpha} + E_4\dot{\alpha} \tag{2}$$

$$\begin{aligned}
\dot{\theta} &= Q\cos\phi - R\sin\phi \\
\dot{\phi} &= P + \tan\theta\,(Q\sin\phi + R\cos\phi) \\
\dot{\psi} &= \frac{(Q\sin\phi + R\cos\phi)}{\cos\theta} \\
\dot{H} &= U\sin\theta - V\sin\phi\cos\theta - W\cos\phi\cos\theta \\
\dot{x} &= U\cos\theta\cos\psi + V(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + W(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\
\dot{y} &= U\cos\theta\sin\psi + V(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + W(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)
\end{aligned} \tag{3}$$

Equation 1 represents the coupled structural and flight dynamics motion. The vector ϵ represents the deformation of each structural element. Linear and rotational speeds are represented by β. M_{ij} represents the mass matrix (which can change along time due to structural deformations); $C_{i,j}$ represents the damping matrix; K_{ij} is the structural rigidity matrix. It's possible to see that rigid body states (represented by β) are inertially coupled with structural states (ϵ), since the mass matrix is not diagonal.

R_F and R_B are the generalized forces that are applied in the airplane. They are obtained from the aerodynamic, gravitational and propulsive forces applied to each structural node. Strip theory is applied, so that aerodynamic forces and moments are calculed using bidimensional models in each node.

Equation 2 represents the additional states needed to describe the unsteady aerodynamic effects, due to the wake. The unsteady aerodynamic modelling applied was proposed by Peters *et al.* (1995).

Equations 3 are needed to describe the airplane position and orientation. They consist in the Euler angles propagation, as a function of rotational rates, and position propagation, as a function of body frame linear speeds. Deduction to these equations are presented by Stevens and Lewis (2003).

3. LINEARIZATION AND STABILITY

Equations 1,2 and 3 consist of a set of nonlinear differential equations. These equations can be linearized around an equilibrium point. The following expression is obtained:

$$M \begin{bmatrix} \dot{\epsilon} \\ \ddot{\epsilon} \\ \dot{\lambda} \\ \dot{\beta} \\ \dot{\vec{k}} \end{bmatrix} = N \begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \lambda \\ \beta \\ \vec{k} \end{bmatrix} + Bu \tag{4}$$

where $\vec{k}$ is the kinematics vector. This vector represents the variables needed to describe the airplane position and orientation. In the study of stability it is possible to use $\vec{k} = \begin{bmatrix} \theta & \phi & H \end{bmatrix}^T$ and u represents the control inputs (which can be an aerodynamic surface control and/or engine thrust). M, N and B are constant matrices for a given equilibrium condition.

We can also analyze subsystems of the full airplane, supposing that the motions are uncoupled. In this work, two submatrices are studied:

- **Aeroelastic only system:** rigid body degrees of freedom are neglected. This leads to the following system of equations:

$$M_{aeroelast} \begin{bmatrix} \dot{\epsilon} \\ \ddot{\epsilon} \\ \dot{\lambda} \end{bmatrix} = N_{aeroelast} \begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \lambda \end{bmatrix} + B_{aeroelast}u \tag{5}$$

- **Rigid body only system:** flexible degrees of freedom are neglected:

$$M_{RB} \begin{bmatrix} \dot{\beta} \\ \dot{\vec{k}} \end{bmatrix} = N_{RB} \begin{bmatrix} \beta \\ \vec{k} \end{bmatrix} + B_{RB}u \tag{6}$$

Notice that $M_{aeroelast}$ and M_{RB} are submatrices of M: $N_{aeroelast}$ and N_{RB} are submatrices of N. It is possible to study the dynamic stability of each system by calculating the eigenvalues of the state matrix, given by: $A = M^{-1}N$, $A_{aeroelast} = M^{-1}_{aeroelast}N_{aeroelast}$ and $A_{RB} = M^{-1}_{RB}N_{RB}$.

The following procedure is used to study the airplane stability in this paper:

1. Start at a given stable speed V_0;
2. Calculate the equilibrium for the given conditions;
3. Linearize the equations of motion: calculate A, $A_{aeroelast}$ and A_{RB};
4. Calculate the eigenvalues of each state matrix;
5. Determine the maximum real part of the eigenvalues;
6. Increase speed and repeat from step 2.

By determining the maximum real part of the eigenvalues of each state matrix, we can determine the instability speed (once an eigenvalue has positive real part, the system is unstable). The immaginary part of these eigenvalues gives us the frequency associated with the unstable mode. Additionally, it is possible to determine the modal shape associated with the unstable mode by calculating the eigenvectors of the state matrix.

4. FLYING WING DESCRIPTION

Patil (1999) suggested a large aspect ratio cantilevered wing to study aeroelasticity of highly flexible structures. Patil's goal was to verify how large structural deflections affects the flutter speed and frequency of this wing. It concluded that, for this wing, studying the dynamic stability around a geometrically nonlinear equilibrium point would change the flutter speed from 32 m/s to 22 m/s (when compared to the usual aeroelasticity results, which considers an undeformed shape).

In this work, an airplane which uses a very similar wing is studied. It consists in a flying wing, as presented in Figure 1. The (half) wing properties are presented in Table 1. An engine is located in the wing root.

A concentrated mass representing the engine and payload is located in the central wing section. It is located in front of the leading edge, so that the gravity center is located in the 20% of chord position relative to the leading edge (thus ensuring static stability of the airplane). The system stability will be studied considering three different concentrated masses: 10 kg, 12 kg and 15 kg.

Table 1: Wing properties - Patil (1999).

Length	16 m
Chord	1 m
Mass per unit lenght	0.75 kg/m
Elastic axis position (relative to leading edge)	50% chord
Gravity center position (relative to leading edge)	50% chord
Rotational inertia I_{11}	0.1 kg.m^2/m
Inertia I_{33}	0.1 kg.m^2/m
Torsional rigidity $K_{22} = GJ$	1×10^4 N.m^2
Flexional rigidity $K_{33} = EI_{yy}$	2×10^4 N.m^2
Flexional rigidity $K_{44} = EI_{zz}$	4×10^6 N.m^2
Damping coefficient c	0.01%
Flight altitude	20 km ($\rho = 0.0889 kg/m^3$)

5. RESULTS

The aeroelasticity results presented by Patil (1999), commented in the previous section, have shown a large difference between the flutter speed considering a deformed cantilevered wing, when compared to the study of an undeformed wing. Since the equilibrium deformation in flight will be different from the deformation of the cantilevered wing, one would expect different results for the flutter velocity in this case. Furthermore, the rigid body degrees of freedom should influence the stability: they can cause a direct instabiliy, if the phugoid or short period is unstable, for example; or they can cause instability due to coupling with aeroelastic modes.

This work has the goal of studying the free flight motion of the flying wing. To do this, the eigenvalues of the state matrix were studied for an interval of speeds. Three different airplanes were studied, considering a concentraded mass of 10 kg, 12 kg and 15 kg.

Figures 3, 4 and 5 presents the largest real part among the eigenvalues of three state matrices:

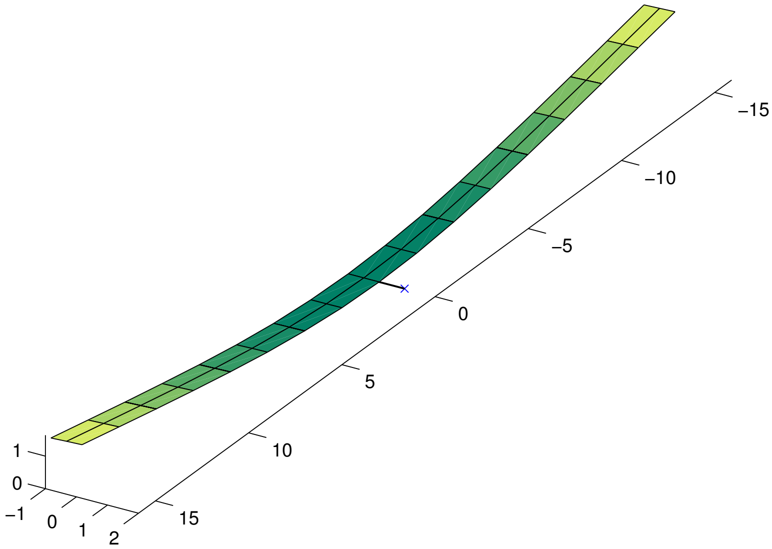

Figure 1: Airplane studied: Flying wing.

- Full matrix A (free flight): including structural dynamics, aerodynamics and rigid body states;
- Aeroelastic matrix $A_{aeroelast}$: including only structural dynamic and aerodynamic states (rigid body states neglected);
- Rigid body states: only rigid body states (structural dynamics and aerodynamic states neglected).

See that the three matrices are linearized around the free flight equilibrium point. Studying the eigenvalues of the aeroelastic matrix means studying a cantilevered wing, but with equilibrium deflections of a free flight equilibrium. Studying the rigid body eigenvalues means neglecting the aeroelastic modes, but considering that the linearization is around the equilibrium of a deformed wing.

Figure 3 shows the results for a concentrated mass of 10 kg. It is possible to see that for low speeds, the full flight motion of the airplane is unstable due to phugoidal motion (this instability is present also in the rigid body eigenvalues). For higher speeds, aeroelastic instability appears. In this airplane, the full matrix instabilities are very similar from the submatrices (aeroelastic and rigid body matrices).

For the airplanes with larger concentrated masses, a different comportament is seen. In the case of 12 kg concentrated mass (Figure 4), the full matrix instability happens at speeds lower than aeroelastic flutter. This instability is related with a coupling between rigid body and aeroelastic motions. This coupling can be seen while observing the modal shape related to this unstable mode (Figure 2). It is possible to see that this modal shape presents both structural deformation and rigid body translation (the flying wing is deslocaded in z axis and rotated in x axis).

In the case of a 15 kg concentrated mass (Figura 5), the free flight instability happens at speeds even lower than classic flutter.

Table 2 presents the results of speed and frequency of instability for each of the three airplane configurations. It is possible to see that for the heavier configurations, the frequency of instability is slower than aeroelastic flutter frequency (actually, these frequency are much closer to rigid body frequencies).

Table 2: Speed and frequency of instability for the flying wing.

Fuselage mass	Free flight		Aeroelaticity	
	V (m/s)	f (rad/s)	V (m/s)	f (rad/s)
10 kg	30.5	12.8	30.5	12.8
12 kg	26.9	4.8	30.9	11.5
15 kg	15.0	3.0	31.4	9.9

Figures 6 and 7 shows the dynamic response after an elevator doublet input. The simulations were performed by integrating the nonlinear dynamics considering the 12 kg airplane. Two different speeds were simulated: 25 m/s and 28 m/s. As predicted by the stability analysis, at 28 m/s we can see that the oscilations are unstable.

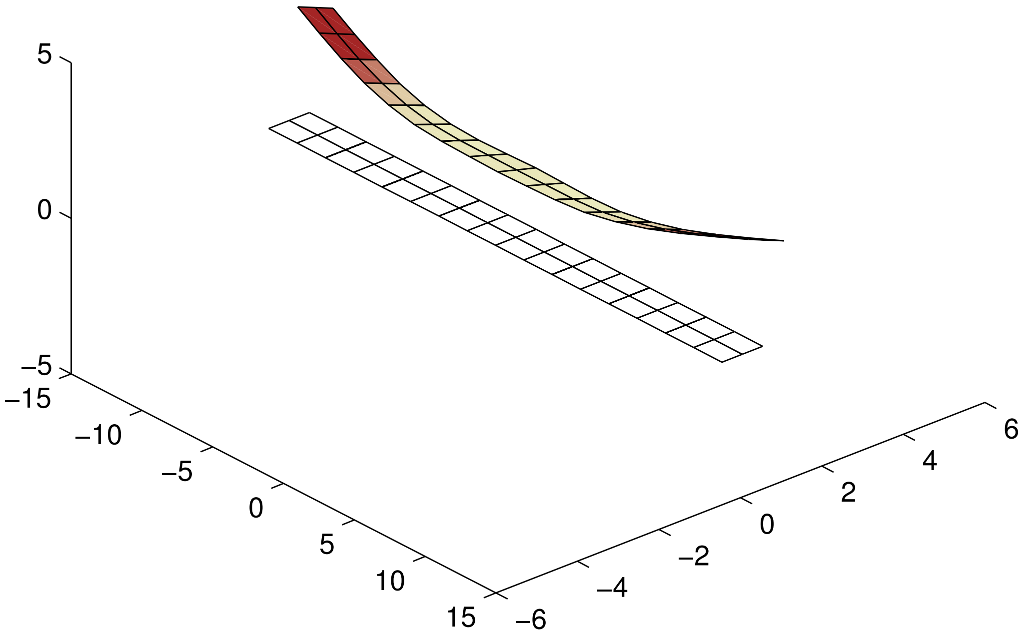

Figure 2: Modal shape related to the unstable mode.

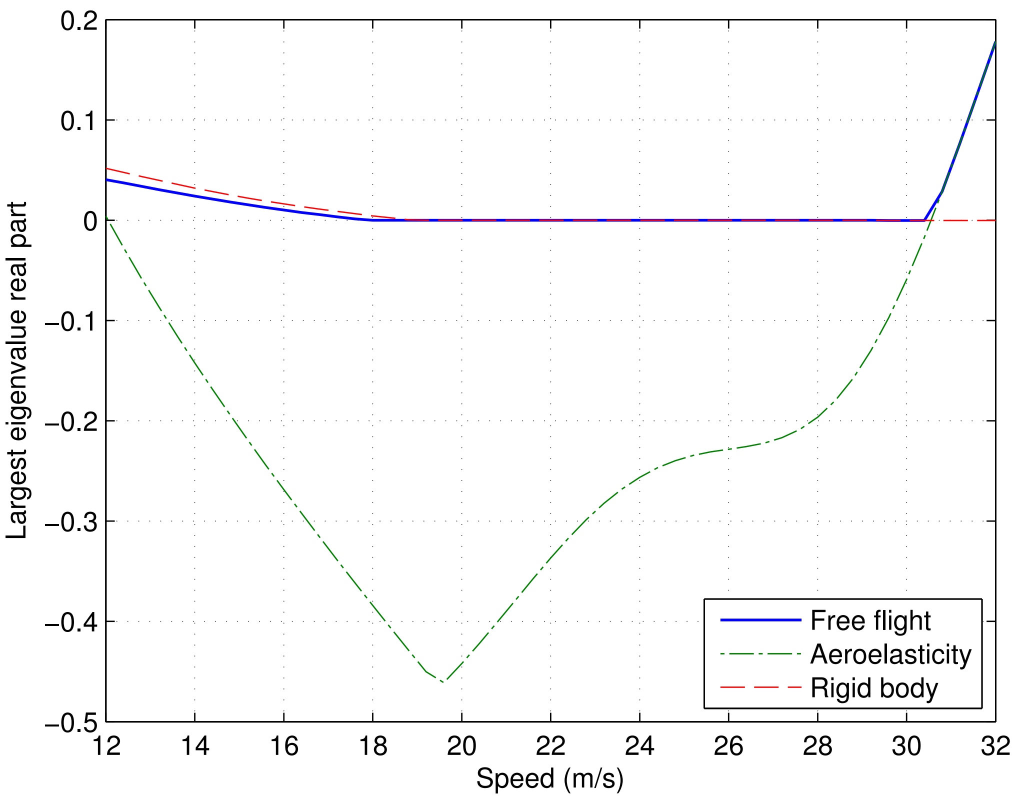

Figure 3: Largest part of the eigenvalues as a function of flight speed (10 kg concentrated mass).

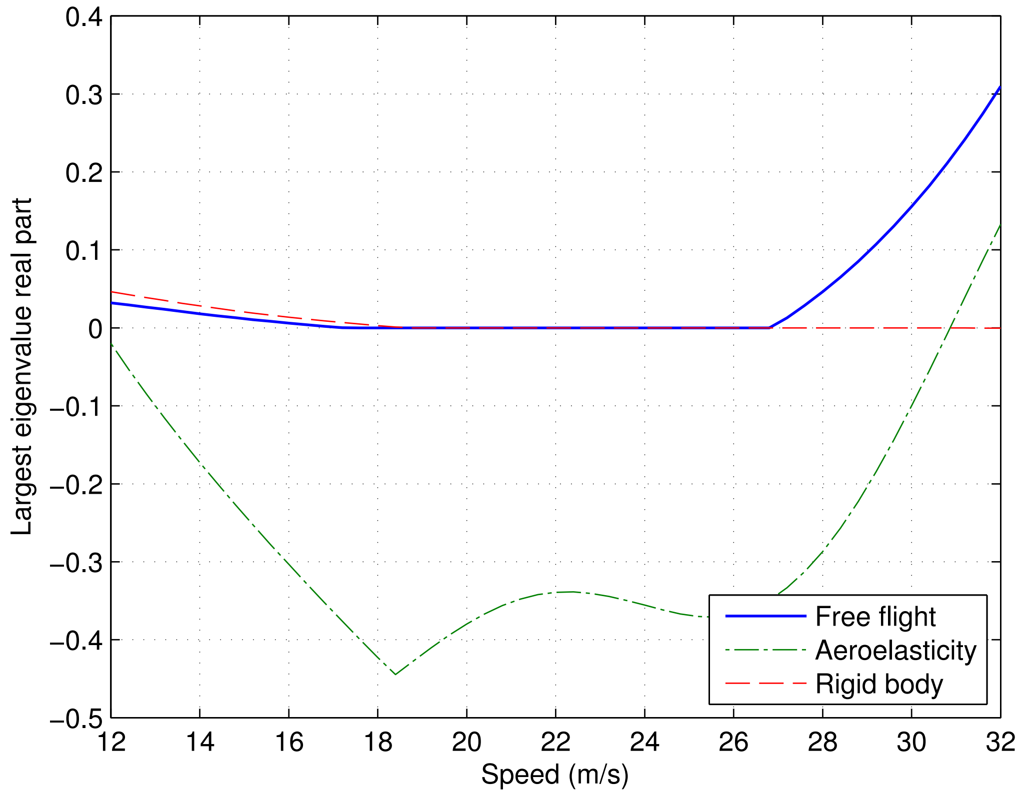

Figure 4: Largest part of the eigenvalues as a function of flight speed (12 kg concentrated mass).

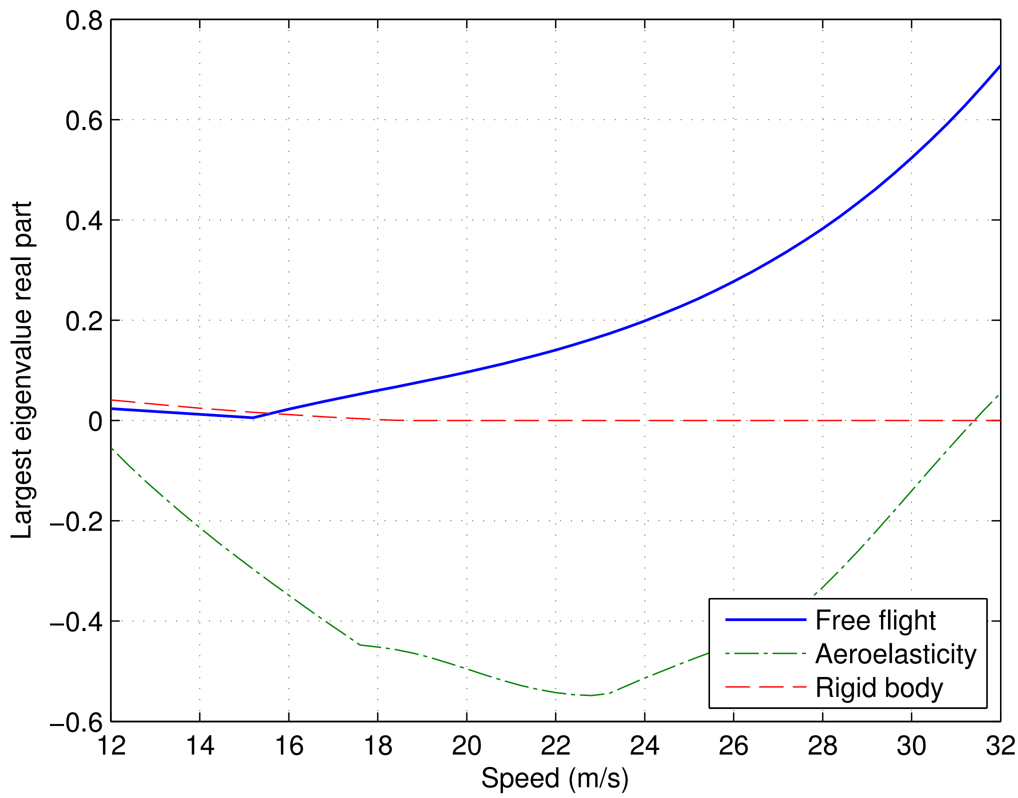

Figure 5: Largest part of the eigenvalues as a function of flight speed (15 kg concentrated mass).

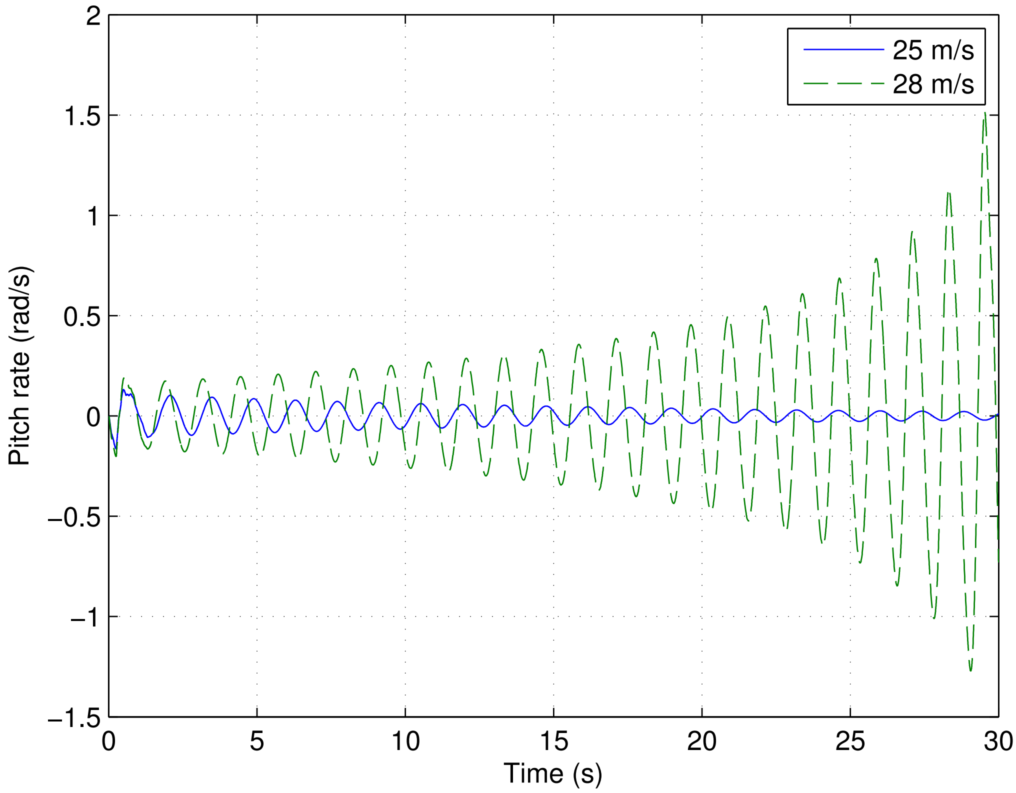

Figure 6: Pitch rate response after a doublet elevator input.

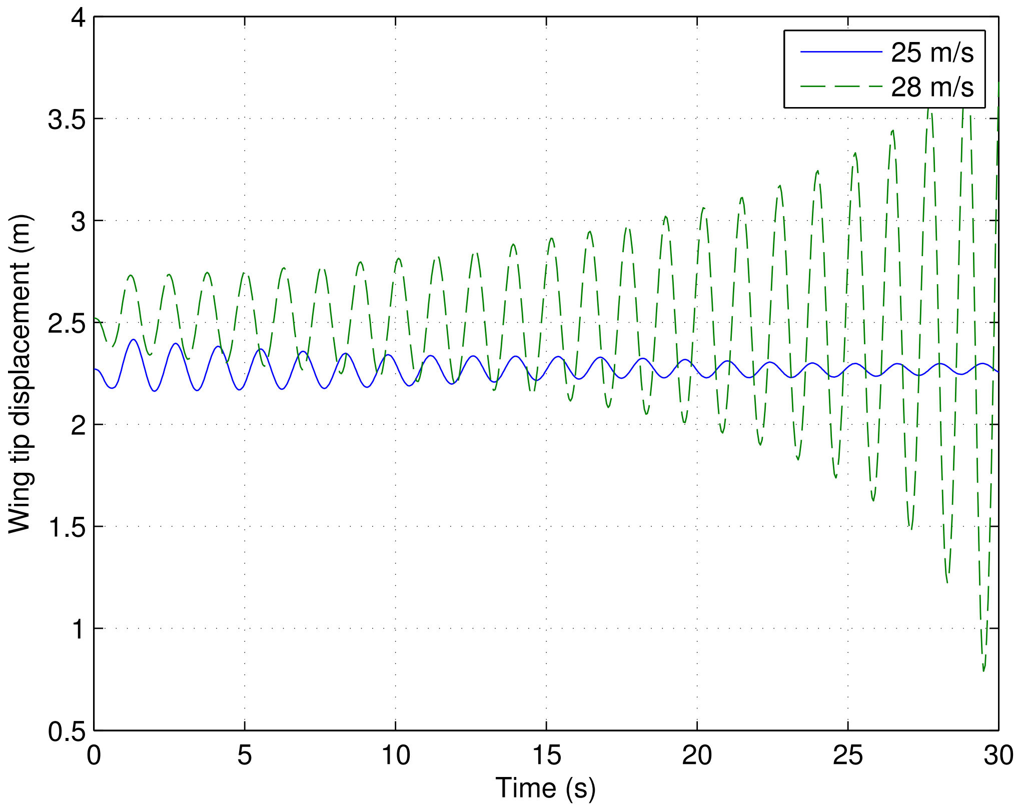

Figure 7: Wing tip response after a doublet elevator input.

6. CONCLUSIONS

The studies presented in this paper have shown that additional care must be taken when studying the stability of highly flexible airplanes. The flutter speed obtained by classical aeroelastic approaches may not be appropriate due to two main reasons:

- The large structural deflections can change the forces distribution around the airplane, changing the flutter speed (meaning that a structural model that allows large deflections must be implemented);
- The rigid body and aeroelastic modes coupling might lead to slower instability speeds (meaning that a flight dynamics model that includes structural dynamics interferences must be implemented).

7. REFERÊNCIAS

Bisplinghoff, R., Ashley, H. and Halfman, R., 1955. *Aeroelasticity*. Addison Wesley, Cambridge, MA.

Brown, E.L., 2003. *Integrated Strain Actuation In Aircraft With Highly Flexible Composite Wings*. Dissertation for a doctoral degree, Massachusetts Institute of Technology.

Hodges, D., 1990. "A mixed variational formulation based on exact intrinsic equations for dynamics of moving beams". *International journal of solids and structures*, Vol. 26, No. 11, pp. 1253–1273.

Patil, M.J., 1999. *Nonlinear Aeroelastic Analysis, Flight Dynamics, and Control of a Complete Aircraft*. Dissertation for a doctoral degree, Georgia Institute of Technology.

Peters, D., Karunamoorthy, S. and Cao, W., 1995. "Finite state induced flow models. i: Two-dimensional thin airfoil". *Journal of Aircraft*, Vol. 32, No. 2, pp. 313–322.

Ribeiro, F.L.C., 2011. *Dinamica de voo de aeronaves muito flexiveis*. Master of science thesis, Instituto Tecnologico de Aeronautica.

Shearer, C.M., 2006. *Coupled Nonlinear Flight Dynamics, Aeroelasticity and Control of Very Flexible Aircraft*. Dissertation for a doctoral degree, The University of Michigan, Ann Arbor.

Stevens, B. and Lewis, F., 2003. *Aircraft control and simulation*. Wiley-Interscience, Hoboken, New Jersey.

Su, W. and Cesnik, C., 2010. "Nonlinear aeroelasticity of a very flexible blended-wing-body aircraft". *Journal of Aircraft*, Vol. 47, No. 5, pp. 1539–1553.

Su, W., 2008. *Coupled Nonlinear Aeroelasticity and Flight Dynamics of Fully Flexible Aircraft*. Dissertation for a doctoral degree, The University of Michigan, Ann Arbor.

Weisshaar, T.A., 1995. *Aircraft Aeroelastic Design And Analysis*. School of Aeronautics and Astronautics. Purdue University - Notas de Aula, West Lafayette, Indiana.

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